A STUDY
ON
EXCESSIVE PRECIPITATION

A THESIS
PRESENTED TO THE FACULTY OF THE GRADUATE SCHOOL
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He completed the six-year course of middle school in three years 1925 to 1927 inclusive. From 1927 to 1928 he did a preparatory work for the college course.

From 1928 - 1932 he completed the four year course in Tangshan College of Civil Engineering, Tangshan, Hopeh, China. In July 1932, he received the degree of Bachelor of Science in Civil Engineering and was elected a member of the Phi Tau Phi Honorary Scholastic Society. His published researches on structural engineering in the College Bulletins No. 3,4, and 6 are entitled "A Study on the Gap Theory of Concrete Aggregates", (with C.N. Li and C. Yeh), "Stresses due to Arch Shortening - with an Exact Method of Solution Proposed by the Author", and "Stress Distribution in Riveted Joints, Proposing an Exact Method of Investigation". Other researches are scattered in various magazines.

In 1933, he built the Rimer Kan-Shan Bridge along the Hong Kong Railway, China.

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PREFACE

The solution of an engineering problem may be divided, in general, into three steps: (1) the determination of the fundamental data; (2) the design of the required structures; and (3) the construction of the project. It often happens that the first step is the most difficult, requiring a high type of engineering judgment and broad experience. The selection of the live load system for a bridge, or the determination of the capacity of a water supply system, is a problem that, as a rule, cannot be definitely solved. Particularly in those fields of hydraulic engineering it is the hydrological data that makes engineers to study in a measure besides the engineering science proper.

The ultimate engineering purpose of studying hydrology is to know the discharges of the river and particularly the maximum value in the problems of flood control. To obtain this, there are usually two ways: the direct method and the indirect method. The direct method is the systematic stream gaging by which a rating curve is obtained. Other greater discharges of rarer happenings can be estimated by the statistical method.

The indirect method consists of two parts: the determinations of excessive precipitation over the drainage area, and of the run-off coefficient to obtain the discharge. It is the former part of the indirect method that will be treated in this volume.

The author attempts to introduce the statistical method in the solution of such a problem throughout the development of the following statements. In Section I his opinions on the statistical theories are first presented. Sections II and III give his suggestions on the methods of analyzing flood problems by collecting and interpreting rainfall data.

Throughout the development of this whole volume, one general principle was always borne in mind: "avoiding as much as possible unnecessary speeches that are not his initiative opinions." 免除廢話" This is due to several reasons. It always appears to the author that the right attitude of a writer should be honest. He should express what is his own idea in a simple and explicit manner, quoting others only when it is necessary. Purposely expansion of statements without equal increase of thoughts wastes peoples time to read and makes them dislike the matter involved. No attempt is here made to write a textbook on this subject for elementary students. Readers are supposed to be already acquainted with the references given in each of the following articles. Another reason is that Professor F.J. Seery, the author's major professor, lost one eye when he was a youngster. In every test or examination besides the several questions, he always writes on the blackboard: "Write briefly and fully, please!" So it would not be virtuous to the author to write any vague statements that would waste any of his major professor's energy.

The author wishes to take this opportunity to acknowledge his indebtedness to F. J. Seery, Professor of Hydraulic Engineering and R. A. Mordoff, Professor of Meteorology, for their suggestions and help in improving this volume.

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A STUDY OF EXCESSIVE PRECIPITATION

IN THREE SECTIONS

Section I - Author's Criticisms and Suggestions on the Theories and Applications of Frequency Curves.

Section II - Excessive Precipitation on Small Drainage Basins.

Section III- Excessive Precipitation on Large Drainage Basins.

S Y N O P S I S

Section I

Author's Criticisms and Suggestions on The Theories and Applications of Frequency Curves.

In this section, the author expresses his opinions on the skew frequency curves as the basis of statistical method. Starting from the fundamental foundations–the theory of probability and method of moments, upon which the method is based, through the development of the theory of frequency curves, and its application to the engineering field, critical criticisms are briefly made throughout. A thorough discussion is, however, beyond the scope of this little volume.

I - HISTORICAL NOTE. A concise history of the development of the theory and the recent applications to the engineering field is compiled herein.

II - THE BASIS OF STATISTICAL ANALYSIS. The article treats briefly on how the statistical analysis is developed from the theory of probability and the method of moments.

III - ON THE METHOD OF MOMENTS. The author critisizes the method as being only one of the available methods although broadly employed. Two facts that demands consideration in practical work are mentioned.

IV - ON THE METHODS OF ADJUSTMENTS OF DATA IN THE METHODS OF MOMENTS. The author proves the fallacy of the ordinary method of adjustments of data when the statistics constitute a collection of isolated terms. The method is broadly employed by statisticians but has never been discussed for its fitness so far.

V - ON THE TYPE OF GENERALIZED PROBABILITY CURVE TO BE ADOPTED. Author's general criticisms on the great controversy are mentioned with the conclusion that the theory of frequency curves have not been sufficiently correlated to the mathematical theory of probability to secure a logical foundation on the laws of probability.

VI - ON THE ALLEN HAZEN'S METHOD. The author raises objections against the Hazen method upon the reason of its absence of "foundation on the theory of probability", and concluded that it is simply an empirical method of fitting the observations which is valueless.

VII - ON THE H. H. GOODRICH'S METHOD. Some criticisms and also comments are given about the method.

VIII - ON THE J. J. SLADE'S FUNCTION. Some discussions on Slade's theory and objections against the Slade's objections directed against the Pearsonian theories are given.

IX - APPROXIMATE METHOD OF APPLICATION TO GRAM - CHARLIER SERIES. Tolley's derivation is introduced and the author's plotted corresponding duration curves are presented.

X - THE TYPES BEST ADAPTED FOR STUDY ON HYDROLOGICAL DATA. Discussions on the general requirements in the statistical analysis for hydrological studies are given and the best types of frequency curves available recommended.

AUTHOR'S CRITICISMS AND SUGGESTIONS ON THE THEORIES AND APPLICATION OF FREQUENCY CURVES.

I. HISTORICAL NOTE

Among the pioneer researches on the mathematical theories of probability, the immortal work of the great Laplace, "Theorie analytique des Probabilities", a work which despite its age remains the most important contribution to our present day. Formerly, the Gaussian normal curve of error was held by the older school of statisticians to be sufficient to represent all statistical frequencies, and actual observed deviations from the normal curve were attributed to the limited number of observations. Through the original memoirs of Lexis and the investigations of the Thiele, the fallacy of such a dogmatic belief was finally shown. The researches of Thiele, and later of Pearson developed afterwards, the theory of skew curves of distribution of observation. As recently as 1905 Charlier finally showed that the whole theory of errors or frequency curves may be brought back to the principles of Laplace.

During the last twenty years the research work in the theory of probability has received a new impetus through the labors of the English biometricians under the leadership of Karl Pearson; the Scandinavian statisticians Westergaard, Charlier and Kiaer, the German statistical school under Lexis, and the brilliant investigations of the Russian School of Statisticians. Each group of these investigations have, however, moved along its own particular lines, as, for instance, the English schools have mostly limited their investigations to the field of biology. That unfortunately the great number of contributions by scholars of the remaining countries has not been made familiar to the great majority of English readers seems to be on account of the varieties of languages and difficulties in advanced mathematics to the ordinary readers.

The introduction of the probability methods to engineering problems is not new. In the discussion of a paper by W. E. Fuller, M. Am. Soc. C.E., entitled "Flood Flows" in Transactions Am. Soc. C.E. Vol. LXXVII (1914), p. 670. G. B. Pillsbury, Mem. Am. Soc. C.E., considers the application of the normal law of error to flood flows. In his closing discussions on this paper, Mr. Fuller states: "The values (of flood flows) follow, not the curve of normal probabilities, but what is called a skew curve."

In a paper on "Storage to Be Provided in Impounding Reservoirs for Municipal Water Supply" Trans.A. C. E. 78, p. 1539, Allen Hazen, M. Am. Soc. C.E., makes a practical application of probability methods to storage problems, using the normal law of error with some modification.

In a paper on "The Probable Variations in Yearly Run-off as Determined from a Study of California Streams", Transactions Am. Soc. C.E., Vol. LXXXIV (1921) p. 191, L. Standish Hall, Assoc. M. Am. Soc. C.E. gives an empirical method for determining the unsymmetrical probability curve, together with an excellent discussion of the relation of the theory of probability to stream flow. studies.

In the same discussion, Mr. Hazen states: "For his own use, to aid in drawing at the ends, the writer is now using a series of factors for curves with different degrees of skew. These factors correspond to those which he used in 1913, but the scheme has been developed and extended to cover varying degrees of skew."

The main scheme of Hazen's method is contained in his book "Flood Flows".

In a paper on "Theoretical Frequency Curves and Their Application to Engineering Problems" in Transactions Am. Soc. C.E. Vol. LXXXVII (1924) p. 142, H. Alden Foster, Assoc. M. Am. Soc. C.E., presents Elderton's Mathematical Analysis of Frequency Curves in a brief outline and shows the application of the formulas to engineering problems.

In a paper on "Straight Line Plotting of Skews Frequency Data" Transactions Am. Soc. C.E., Vol. 91, (1927) p. 1, R.D. Goodrich, M. Am. Soc. C.E., presents some equations derived by an entirely empirical process, from which he designs skew frequency paper for use of plotting any hydrological statistics.

Recently in a paper on "An Asymmetrical Probability Function" Proceedings, Am. Soc. C.E. Vol. 60, No. 8, Part 1 (Oct. 1934) J. J. Slade, Jr. Esq., presents the Charlier's "Logarithmic Transformed Frequency Series" and shows the application of the formulas to hydrological problems.

The writer's attention was brought to this subject by a study of the ancient and modern mathematical theories of probability and also the papers by engineers on the application of these to engineering fields afore-mentioned.

II. THE BASIS OF STATISTICAL ANALYSIS

The fundamental basis of statistical method is established on the theories of probability. These theories begin with the assumption of the given conditions for happening and failing of events. For instance, if the condition is that the chance of each single event happening is uniform, and the problem is to find the probabilities of its happening once, twice, and so on out of some total number of trials, then the "Binomial Series" will be able to represent these probabilities that arise under such mentioned conditions. Another series that occurs is known as the "Hyper-geometrical". Defenders of this series claim that it arises in the way that most of the common events happen. These fundamental differences in the very theory at the outset will yield various forms of formulas of probability which are so called "graduation formulas".

With such an equation of probability established, the next question naturally arises in the method of fitting it to the vast amount of data available. The method of moments is a general method of finding the constants in a formula suitable to a particular statistical example, and it consists of equating the values of $\Sigma f(n)$ x n (which is called the nth moment, and is summed for all values of n that occur) to similar expressions obtained from the graduation formula. These latter expressions will be algebraic, and simultaneous equations which have to be solved in order to find the aruthmetical constants.

The equations from these mathematical treatments are considered as representing generally the series of given data, from which the magnitude of observation for any probability of happening can be picked out.

III. ON THE METHOD OF MOMENTS

The method of moments has been broadly employed by the mathematical statisticians in the purely theoretical development of the basic formulas as well as the practical work of fitting a given series of datd. It consists of equating the values of $\sum f(n) \times n^t$ to similar expressions obtained from graduation formula. It is simply one of the available methods, but is not the only method available. The mathematicians have used it but they have not shown or proved that they have to use it or that it is the only best method. In Biomatrika, Vol. I, pp. 367, Professor Karl Pearson has shown that the method can be expected to give very good results, and yet he does not prove that it should give best results. Since it is the basis upon which the theory of frequency curves lies, the problem of statistical analysis is, at the outset, entirely questionable.

The situation here is this: In the formation of observation equations, the number of observations i.e., the number of equations, is always greater than the number of unknown parameters or constants in the graduation formula to be determined by the observations. In other words, we find the problem over-determined. And the mathematicians attempt to introduce the method of moments to solve it. Of course this is the only one method, but not the only method as we can have any treatments to take account of the surplus data. Evidently in attacking such a problem, the method of least squares* will be the most rational, though might not the best adapted. It is the only method that is logically sound.

However, the method of least squares entails too much labor in its practical application. The number of data given means the number of observation equations or the number of conditions equations, yet the labor

* The reader should not be mixed up with the normal law of error adopted as a type of frequency curve which is another problem independent of the one at hand.

of forming normal equations from them will be considerable. In Arne Fisher's "Mathematical Theory of Probabilities" p. 325, an example of six observations is given, in which the work if already terrible. The author just wonders why not a twenty observation example (which number is not uncommon in problems of rainfall and run-off) is offered. This very drawback offsets the sound logic of the method and makes it out of consideration.

Next the method of moments seems to be the only one preferable. Mr. Arne Fisher states in his "Mathematical Theory of Probabilities": "In the purely theoretical development it matters but little whether we use moments or least squares in the expansion of a frequency function in a series; a fact which is readily seen from our previous demonstrations." In the purely practical work, however, we have two facts to consider: (1) The method of moments works exclusively with areas expressed as definite integrals which are often difficult to determine in extremely skew distribution. And it is only by successive approximations that a plausible result can be obtained and which will be discussed in the next subject of criticism. (2) Unless the observations are very numerous, it is almost hopeless to compute the moments of higher than the fourth, because of the very large errors arising from random sampling. This is clear when we remember that extreme values of the data are multiplied by the highest numbers and their powers.

As to the practical method of calculating the moments from the statistics, there are two ways available (See Chapter IV. Frequency Curves and Correlation, by W. Palin Elderton): (1) By multiplying the frequency by appropriate values of n^t; or (2) By Mr. G. F. Hardy's summation method. The latter in combination with the ingenious method of transformation of the reference vertical should be highly recommended because of its convenient application.

IV. ON THE METHODS OF ADJUSTMENTS OF DATA IN THE METHODS OF MOMENTS.

Since the moments from the graduation formula must generally be found by means of the integral calculus, while those from the statistics or given data are found by summation, the latter have to be adjusted before the equations for obtaining the constants can be correctly formed. The adjustments depend upon two cases: (1) When the statistics form a system of isolated terms or ordinates, i.e., quantities of Y in the frequency curve $Y = f(x)$. The adjustment is made by Eq. V, p. 27 in Elderton's Frequency Curves and Correlation. (2) When they are a system of areas but the moments are calculated by assuming the areas to be concentrated at the middle points of the bases, i.e., the statistics are given in quantities of P(= probability) in the duration curve where $P = \int Y\,dx = \int f(x)\,dx$.

The Sheppard's adjustments will give good results, if there is high contact* at both ends of the curve.

It is the former method of adjustments probably originated by Prof. Karl Pearson of the University College, London, that the following criticism is offered.

* "High Contact" means a curve closed very quickly to its asymptote.

It is evident that the area under the curve is expressed by $\int Y_x \, dx$, (the derivation of graduation formula must be found by integral calculus) while the area under the steps are formed upon ordinates spaced at unit distance apart $= \Sigma (Y_x \, x1) = \Sigma \, Y_x$. (The moment from statistics are formed by summation). Therefore, the latter ordinates Y_x should be so corrected that after summation it will be equivalent to the former. Elderton gives an equation for the adjustment in his "Frequency Curves and Correlation" p. 27, as follows: (The derivation is correct)

$$\int_{-1/2}^{n-1/2} Y_x \, dx = \frac{\text{[illegible]}}{5760} \{ 6463 \, Y_0 + 4371 \, Y_1 + 6669 \, Y_2$$

$$+ 5537 \, Y_3 + 5760 \, (Y_4 + Y_5 + \ldots . Y_{n-6} + Y_{n-5})$$

$$+ 5537 \, Y_{n-4} + 6669 \, Y_{n-3} + 4371 \, Y_{n-2} + 6463 \, Y_{n-1}$$

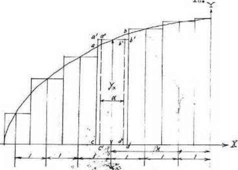
which means that we should multiply the first and last ordinates by $\frac{6463}{5760}$ (= 1.1220485),

the second and the last but one by $\frac{4370}{5760}$ (= 0.7588541),

the third and the last but two by $\frac{6669}{5760}$ (= 1.1578127),

the fourth and the last but three by $\frac{5537}{5760}$ (= 0.9612847),

and leave all the other ordinates unaltered.



Now consider any area abcd under the curve which represents $\int_{x-1/2}^{x+1/2} Y_x \, dx$. The area under the corresponding step is $a'b'cd_o$ and equals $Y_x \times 1$. It should be so adjusted that

$$Y_x \cdot \alpha_x = \int_{x-1/2}^{x+1/2} Y_x \, dx.$$

where α_x is an appropriate coefficient from the above equation. Or,

Area a" b" c' d' = Area a b c d

But here the ordinate Y_x situates at the midpoint of c' d', (and cd) while not at the center of gravity of the area a b c d. (Unless the curve is a straight line which never happens for probability functions). Therefore, for this area a b c d,

$$(\quad {}_x Y_x) \,.\, x \quad \int_{x - 1/2}^{x + 1/2} X Y_x \, dx$$

and,

$$\overset{n - 1/2}{\underset{-1/2}{}} (\quad {}_x Y_x) \,.\, x \quad \int_{-1/2}^{n-1/2} X Y_x \, dx$$

Similarly,

$$\overset{n-1/2}{\underset{-1/2}{}} (\quad {}_x Y_x) \, x^r \quad \int_{-1/2}^{n-1/2} X^r \, Y_x \, dx$$

except when r = o, for which case the corrections x are deliberately made.

In the method of adjustments given by Elderton, the same set of corrections is used for all powers, which has been disproved as above. It might be well to mention here that this kind of adjustment is not correct for any power of moment except for area or zero moment. The different adjustments which should be made for each particular power of moment, can evidently be derived without much difficulty.

V. ON THE GENERALIZED PROBABILITY CURVE TO BE ADOPTED.

A Great Controversy – It is a big unsolved problem yet as to just what type of a generalized probability curve should be universally adopted for all conditions of events. Standing on their own side, advocates of each of the two theories state their own merits and defects of their opponents.— **The Pearsonian Curves and the Gram-Charlier Series.**

The Pearsonian Method is based upon the "Hyper-geometrical series" as its foundation of probability theory. The latter is embodied in the particular case as follows: "The chances of getting r, r – 1, ... 0 black balls from a bag containing pn black and qn white balls when r balls are drawn, are given by the successive terms of the series: –

$$\frac{{}^{pn}C_r}{{}^{n}C_r}\left\{1 + \frac{r\ qn}{pn - r+1} + \frac{r.\ r-1}{2\,!}\ \frac{qn(qn-1)}{(pn - r+1)(pn - r+2)} + \ldots\right\}$$

Next the method assumed the two obvious characteristics of frequency distributions, i.e. (1) High contact at ends of distribution: $Y = 0$ when $\frac{dy}{dx} = 0$; and (2) A point of maximum ordinate in the curve called mode : $\frac{dy}{dx} = 0$, when $x = -a$.

It follows: $\frac{dy}{dx} = \frac{Y\,(x+a)}{F(x)}$

Another assumption is that the function $F(x)$ can be expanded by Maclaurin's theorem, so that

$$\frac{dy}{dx} = \frac{Y\,(x+a)}{b_0 + b_1 x + b_2 x^2 + \cdots\cdots}$$

Byusing the general term of the hyper geometrical series, Pearson proves, (this step is strictly logical)

$$\frac{dy}{dx} = \frac{Y\,(x+a)}{b_0 + b_1 x + b_2 x^2}$$

Criticisms on the Pearsonian theory are mostly directed against the insufficient theoretical basis of the system. The objection lies on the fact that it is based on the empirical characteristics of frequency distributions. To the author, however, it seems that other series like Gram-Charlier are all empirical or starting with an assumption. It is interesting to note that a paper read before the Statistical Society of England, Professor F. Y. Edgeworth, who has himself suggested other methods, points out that Pearsen's Generalized Probability Curve appears more justifiable the longer its philosophic basis is subjected to criticism.

Frye's explanation of the Pearson Curve is rather instructive: (In his "Probability and Its Engineering Uses" p. 344) "This foundation consists of the observation that, in a certain approximate sense the Normal Law, the Binomial Law, the Poisson Law and the law of repeated dependent trials all satisfy the differential equation $\frac{1}{P} \times \frac{dP}{dx} = \frac{a + x}{b + cx + dx^2}$ for some sets of values of the constants a, b, c, d."

The Gram-Charlier Series - starts from a basis which is claimed as much sounder logically than is that under lying the Pearson Curves. It consists of the use of the infinite series $y = A_0 F(x) + A_3 F'''(x) + A_4 F''''(x) + \ldots\ldots$ where $F(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-b)^2/2\sigma^2}$ rather than a closed expression. The curve has been given by Edgeworth, Camb. Phil. Trans. Vol. XX., pp. 36-65,113-141.

Various merits and demerits have been raised on this series which will not be given here.

The point to be noticed, however, is this: The form of the function $F(x)$ is also assumed.

Author's general criticism - All statistical analysis at present are more or less empirical. The two fundamental probability series are the binomial and hypergeometrical.

The more common distribution laws lead to them in a more or less approximate sense are: The Normal Law, the Poisson Law, the Pearson Types, and the Gram-Charlier series. While none of them give an exact solution supporting the perfect theory of probability, the Pearson Types and the Gram Charlier Series may usually offer wide ranges of application to any field of statistics. In considering the convenience in the application to such a theory, personally the author prefers the former (the Pearson Types) since there is no sufficient evidence to support the latter's more improved theoretical basis.

Like the present situation of the world's spiritual civilization, the mathematical theory of probability with its application to the theory of frequency curves has not been developed far enough even to support itself on a logical foundation. The future solution of this problem should still explore its way through the means of mathematics as the Great Laplace did. And we, as engineers, have to lay down this burden on the shoulders of the mathematicians.

VI. ON THE ALLEN HAZEN METHOD

The late Allen Hazen* is the first man who used

* Transactions Am. Soc. C.E. Vol. 87 (1924) p. 176.
Hazen: "Flood Flows" John Wiley and Sons.

the factors of skew curves in connection with the study of storage and spillway porblems in California about twenty years ago. He used a number of artifically prepared series of terms whose logarithms follow the normal law of error and called such a series "Logarithmic Probability Series". In cases, in fact in most cases, the coefficient of skew and the coefficient of variation of a set of observations do not correspond with each other as those of the prepared series, which he called "Modified Logarithmic Probability Series". The preparation of such an artificial series is fully shown in Tables VIII, IX, and X of the Appendix of his "Flood Flows".

This method is entirely undesirable. It is purely empirical, with little mathematical basis underlying it at all. It is simply a method of fitting the data by drawing a smooth curve through the scattered points, except that it can fit the data better.

Yet his method is not rigid by itself. Hazen says in his "Flood Flows" p. 57, "On inspecting 109 plottings, 15 were found which, considered in this way, would be better represented by coefficients of skew somewhat different from those reached by the above described method of calculation. The proceduce followed when this condition was observed was to make another plotting based on a lower or

higher assumed value of the coefficient of skew.... ."
This shows that sometimes the curve cannot fit the data well, and another trial should be made.

Since it is nothing more than means of fitting the data, the whole reliability of the resulting frequency and duration equations will depend upon the fitness and the result for which we are looking has a direct bearing on the amount of data available.

As to the dogmatic means of fixing the logarithms of a skew frequency series to be "normal probability", the author may interpretate it as being analogous to the Logarithmic Transformation of the Gram-Charlier Series. It often happens that even if the observations form an extremely frequency distribution, its logarithms will be nearly normally distributed. In fact, this is a matter of definite consequence. Logarithms of numbers will surely show smaller variations than the numbers themselves, so that logarithmic plottings will evidently give more smooth fitting curves. Arne Fisher also said: * "This fact was already noted by the eminent German psychologist, Fechner, and also mentioned by Bruhns in his Kol-

* Fisher: "Mathematical Theory of Probability" p. 237.

lektionsaslehre. But neither Fechner nor Bruhns have given a satisfactory theoretical explanation of the transformation and have limited themselves to using it as a practical rule of thumb.

There is another point on which the author wishes to express his opinion here: Mr. Hazen states: # " ...... for stream-flow data, there are two reasons why mathematical analysis is not advisable. First, the difficulty of applying the methods is too great for some engineers; and, second, and more important, such short-term records as are available are not long enough to serve as a basis for a conclusive determination of the shape of the curve." Hazen's idea, and also of many other engineers, is that the longer the terms of records, the more we can depend upon the theoretical equation of probability, while the shorter the terms, the less will be the theoretical equation reliable. This is wrong - in case of absence of long term records, the more becomes the importance of having a logic equation of sound theoretical basis, so that with a small amount of record available one can still make use of it and obtain good results. On the other hand, if the terms are

Transactions Am. Soc. C.E. Vol. 84 (1921) p. 314.

abundant, and of wider range, equation derived therefrom has much bearing upon the data, which, if correct, should be of logic distribution so that reliance upon the probability theory becomes less important. With such a view in mind, one should be able to visualize the value of possessing an equation standing on the logic theory of probability.

VII. ON THE R. D. GOODRICH'S METHOD.

In a paper on "Straight Line Plotting of Skew Frequency Data", Trans. Am. Soc. C.E., Vol. 91 (1927) p. 1, R. D. Goodrich, through several years study searching for equations that could be used as guides in estimating the magnitudes and frequencies of floods and in extending the equations when found, suggests a set of entirely empirical equations from which he designs skew frequency paper for plotting any hydrological statistics. His general equation of duration curve is embodied in the following function:

$$t = n - ne^{-k\frac{(R-a)^c}{(b-R)^3}} \quad ---- \quad (G)$$

which he admits to be too complicated for general use and gives another three special forms of (G):

$$t = n - ne^{-k(R-a)^c} \text{ - - - - - - - - (A)}$$

$$t = n - ne^{-kR^c} \text{ - - - - - - - - (S)}$$

$$t = n - ne^{-k\left[\frac{R-a}{b-R}\right]^c} \text{ - - - - - - - - (B)}$$

The corresponding frequency curves are respectively as follows:

$$t' = \frac{dt}{dR} = n\,c\,k\,(R-a)^{c-1} e^{-k(R-a)^c} \text{ - - -(A')}$$

$$t' = \frac{dt}{dR} = n\,c\,k\,R^{(c-1)} e^{-kR^c} \text{ - - - - - - (S')}$$

$$t' = \frac{dt}{dR} = n\,c\,k\,(b-a)\frac{(R-a)^{c-1}}{(b-R)^{c+1}} e^{-k\left[\frac{R-a}{b-R}\right]^c} \text{ - -(B)}$$

The most remarkable point in the form of the above function is the striking similarity of equation (S') to Equation (E)*

$$y = c\ Z^p\ e^{-Z}$$

as presented by H. Alden Foster, with R^c in place of Z in the exponent of e. Now since the latter is a transformed form of Pearson's Type IV, Goodrich's Eq. (S') corresponds to it and therefore should attain as sound a mathematical foundation as Pearson's possesses, in spite

* "Theoretical Frequency Curves and Their Application to Engineering Problems", Transactions, Am. Soc. C.E. Vol. 87, (1924), p. 151

of its empirical origin. Furthermore, his ingenious method of plotting data incident to the type of Eq. (3') on the so called "Skew Frequency Paper" would be an energetic impetus to those mathematicians to open a new field through the mathematical means of transforming the complicated Pearson's Equations to such forms as to render the graphical treatment possible. Readers will appreciate the author's comments on Goodrich's influence whenever one day such a method actually occurs. Those versed in the handling of differential equations will be surely aware of such a possibility.

Nevertheless, Goodrich's Eq. (A) and (B) do not possess any form of other Pearsonian Types, although they have similar scopes of application to some of the latter. Consequently, they are empirical as they are, and do not stand on a sound mathematical foundation. Their application will find limitations as Mr. Goodrich himself said: (p. 40, of his paper) "That there are certain limitations and difficulties in applying the writer's equations, especially when both limits of the curve are finite, is admitted."

Mr. Goodrich's "Method of Straight Line Plotting" however, has new significance. As pointed out by the author in Article II, basis of statistical methods consist of (1) the adoption of a reasonable probability theory; and (2) the method of fitting a given set of data to such a deduced equation. The method of moments is simply a process of the latter, but is neither the only one nor the best. Here Mr. Goodrich's suggestion, by ruling the paper according to a theoretical function of probability(whether it is correct or not is another question) following which plottings will trace a straight line, is a new process of fitting the data. But in this process, however, there is no definite rule in weighing any datum along any range of magnitude as the method of moment does. Whether this is its merit or defect we do not know, until we know the manner of the method of fitting by the method of least squares* on graphs. Personally, the author favors it, though he could not prove its propriety.

One of the demerits in the Goodrich method is the troubles encountered in determining the limits a, or b, or either. The type of curve that a set of given data incident to could not be understood until it is plotted on the skew frequency paper. For instance, in Eq. (A) when t is plotted to R, the curve will be

* The reader should not be misled as we are applying the Normal Law of Distribution to the adoption of frequency types.

concave upward if the constant a is positive, and will be concave downward if negative. After the correct value of a is determined by several trials, y and (R-a) will plot on a straight line. Further hazard operations are required in fitting Eq. (B). Such a way of finding the limiting points is, of course, a drawback of the method in comparing with the Pearson's and Foster's mathematical deduction of constants. In case the limit or limits should be assumed from the physical conditions on account of the irregularity of the data, then such extra-statistical constants that calls for the exercise of the engineer's judgment, will be usually so uncertain in a measure as to be contradictory to the limiting tendencies of the theoretical curve.

Another weakness in the method is due to the impossibility of computing the probable error (or mean square error) and coefficient of skewness of the constants in the functions. Those new factors as "Average range", "quartile ratio" suggested by Mr. Goodrich are not equalled in value to the well known factors mentioned. Mr. Alden Foster, has this same opinion. (See Discussions on Goodrich's paper, p. 45).

VIII. ON THE J. J. SLADE FUNCTION

Recently, J. J. Slade, Jr.Esq., presented a paper to the Proceedings of Am. Soc. C.E. Vol. 60, No. 8, entitled: "An Asymmetric Probability Function", on which the author wishes to express his opinion.

Mr. Slade's criticisms on the Pearson Curves is so stated;- "Whereas Eq. (2) (i.e. $\frac{dy}{dx} = \frac{Y\ (x + a)}{F\ (x)}$) may contain all frequency curves, it is quite likely that it also contains many curves not even remotely related to frequency functions. Eq. (3) does not provide a means of distinguishing between them; ....". "Another difficulty with Pearson's set-up is a practical one. . . . Pearson assumed that F(x) can be developed in a Maclaurin series and it is permissible to drop all the terms of the series after the square. This is merely a roundabout and unscientific manner of assuming that F(x) is a quadratic." Mr. Slade in raising these two objections against the Pearson Curves is too dogmatic. He seems not to have realized that Prof. Pearson did prove the perfect correctness of dropping all the terms of the series after the square from the derivation of the general term of the hyper-geometrical series that is his basic theory of probability. (See Elderton's Frequency Curves and Correlation)

Evidently the question is whether the series could be served as a basic theory or not. (See Discussions, Art. V) As to the manner of dropping the subsequent terms, it is rather the chief merit of the Pearson Curves over the Gram Charlier Series in that the former does prove the permissibility of such a manner, while the latter drops the subsequent terms of the assumed infinite series simply by approximation.

Again, although the equation may contain curves not related to frequency functions, but that will not be a drawback to setting such an equation, because it is perfectly general, the parameters of which are to be determined from the set of data. For instance if the desired function is $f(x)$, and the present function $F(x)$ contains $f(x)$ but also other curves such that $F(x) = f(x) \cdot \mathcal{J}(x)$. Every point staisfies $f(x)$ will also satisfy $F(x)$ and the curve of $F(x)$ can represent $f(x)$ as well.

Mr. Slade suggests his type of frequency curve represented by the function:

$$Y = a\, e^{-c^2 \left[\log d\,(x-b)\right]^2}$$

which is defined from $x = -b$ to $x = \infty$. This function presumes its similarity to two well known functions: (1) The Normal Law as its final limit when the coefficient of skewness = 0, and (2) the logarithmically transformed function of Gram-Charlier Series, from which it is entirely different, however. Incidently, his function is assumed or with no perfect bearing on the theory of probability. So it returns finally to the fundmental question as discussed in Art. V. While from this standpoint the author would still prefer the old Pearsonian Types.

It seems that Mr. Slade has not tested the scope of applicability of his proposed function, though he does give an example for comparing the results from his and the Pearson Types. As the function does not appear to be as general at the outset as other well-known functions do, we have to wait for subsequent testimonies.

IX. APPROXIMATE METHOD OF APPLICATION TO GRAM-CHARLIER SERIES

If in the Gram-Charlier Series given in Art. V, the origin is taken at the mean, and if for simplicity, the entire area of the curve is designated as unity, the coefficient A_0 becomes unity, and the equation reduces to

$$Y = F(x) + A_3 F^{III}(x) + A_4 F^{IV}(x) + \ldots\ldots$$

in which $A_3 = -\frac{\mu_3}{3!}$

$$A_4 = \frac{\mu_4 - 3\sigma^4}{4!}$$

and σ = standard deviation = $\sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$$\mu_3 = \frac{\Sigma d^3}{n} - 3\left(\frac{\Sigma d}{n}\right)\left(\frac{\Sigma d^2}{n}\right) + 2\left(\frac{\Sigma d}{n}\right)^2$$

The latter two values are a little different from the ordinary considerations on account of the transformation of ordinates. See Davenport, C.B. Statistical Methods, Ed. 3, 1914. pp. 20-21. for formulas deducing these to true mean.

Although the use of the terms of higher order will, of course, give an equation containing a greater number of constants, and on that account will give a closer approximation to any limited set of observations, but the probable error of the constants derived from the higher powers of the deviations is larger, so that it is justified to reduce the equation to the form,

$$Y = F(x) + A_3 \; F^{III}(x)$$

which becomes,

$$Y = \frac{1}{\sigma\sqrt{2\pi}} \; e^{-\frac{x^2}{2\sigma^2}} \left[1 - \frac{\mu_3}{\sigma^3 3!} \left(\frac{x}{\sigma} - \frac{x^3}{\sigma^3} \right) \right]$$

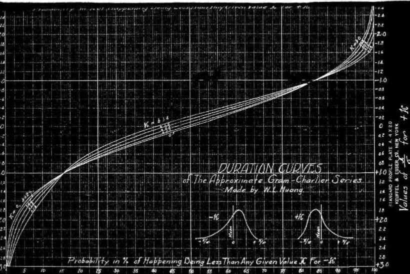
And the equation of duration curve for definite integrals between x_1 and x_2 will be

$$P_{x_1}^{x_2} = \int_{x_1}^{x_2} Y \, dx = \int_{x_1}^{x_2} F(x) \, dx + \left[\frac{1}{\sqrt{2\pi}} \, e^{-\frac{x^2}{2\sigma^2}} \, \frac{K}{3!} \left(\frac{x^2}{\sigma^2} - 1 \right) \right]_{x_1}^{x_2}$$

where $K =$ coefficient of skewness $= \mu_3 / \sigma^3$

The quantity enclosed in the brackets vanishes when $x = \infty$ and it can be evaluated by direct computation for different values of K and σ. The first term $\int F(x)\,dx$ is the duration equation of the normal curve and its value can be obtained from ordinary books of least squares. Tolley, H.R. in "Frequency Curves of Climatic Phenomena" Monthly Weather Review, Nov. 1916, pp. 634-642, has combined the two terms to form new tables.

The author makes a corresponding set of duration curves giving the percentages of the area of the frequency curve to the left of the ordinate whose abscissa is x, for values of k at intervals of 0.4 from k = -1.4 to K = +1.4. By interpolation from these curves, it is possible to find the probability of an observation being greater or less than any value.


DURATION CURVES
of The Approximate Gram-Charlier Series.
Made by W.L. Huang.
Probability in % of Happening Being Less Than Any Given Value X For -K
STANDARD PROFILE PLATE A, 4 X 20
KEUFFEL & ESSER CO. NEW YORK

X. THE TYPES BEST ADAPTED FOR STUDY OF HYDROLOGICAL DATA.

As pointed out in Art. IV and V, the whole theory of statistical method is queer due to its lack of a perfect generalized probability theory and of a sound logic in the use of method of moments. Arne Fisher* states that " .... among the leading mathematicians of the present time there exists no uniform opinion as to the truth of the axioms underlying the theory of probabilities." However, there there was nobody ever raising suspicion against the use of the method of moments so far to the author's knowledge.

From such a standpoint, the author in examining all theories at present available, is inclined to prefer the old Pearsonian Types, both for its more sound mathematical foundation and for its comparatively wider scope of applicability. These types, considered as the best available at present, will always be employed for studies of hydrological data unless difficulties are encountered which give unsatisfactory results in which case the Gram-Charlier Series will be resorted to as a special investigation.

* See page 137, "Mathematical Theory of Probabilities" Fisher.

Those devices and theories as proposed by engineers, biologists, or other specialists for uses in their own fields are not desirable for their general insufficiency in theoretical basis. These include: Allen Hazen's Method, H. H. Goodrich's Method, (only Eq.(8) can sometimes be used) J. J. Slade's Method, and J.C. . Kapteyn's Method, etc.

Various objections against the Pearsonian Types so far raised, do not, as they appear to the author, possess sufficient reasoning.

R. H. Horton, Mem. Am. Soc. C.E., states * "Tolley and others have shown that none of Professor Pearson's curves seem to be very satisfactory for representation of rainfall variation." In fact* when we read Tolley's paper on "Frequency Curves of Climatic Phenomena" we could not find any strong controversy directed against the Pearson's. His slight dissatisfaction with the latter in the paper is probably nothing more than a means of defending his own theory. Professor Edgeworth's paper (See Art. V) read before the Statistical Society of London might serve as a concluding answer to any suspicions on the Pearson's.

In the same paper, Mr. Horton states: "...... most frequency curves utilized by biologists and statisticians, can only be integrated for particular cases. For this reason, it is not wholly satisfactory for engineering work

* Monthly Weather Review, November 1916, pp. 634 - 643.

where integral frequencies, rather than probabilities of individual events, are mostly required. We cannot refuse to admit that Pearson's Method is often difficult to apply, but the tables made by Karl Pearson as "Tables for statisticians and Biometricians", "Tables of the Incomplete Gamma Functions", and "Tracts for Computors No. I - IX," Cambridge University Press do simplify the process very much. Furthermore, Mr. Alden Foster's Works on the solutions of Types I and IV are ingenious and might be extended to develop other types. Again after frequency curves plotted from the Pearson equations, graphical integration may be used to obtain duration curves.

Another important point which might well be explained here is that it is not desirable and yet not practicable to fix the limit or limits of observations by physical conditions. A mathematical equation usually contains besides the functions of physical variations, other solutions which cannot be explained such as negative numbers that mean nothing in some of the ordinary matters, or imaginary numbers whose existence could not be imagined by human minds. For instance, meteorologists and engineers try to fix the lower limit of rainfall equal to zero. But if one imagines this matter one degree factastically, he is likely to think that evaporation and transpiration of water to the atmosphere at times of no rainfall could represent negative values of rainfall. It is really reasonable. Again if such arbitrary limits are once given to a set of observations, it will surely upset the whole theory of frequency curves by the introduction of such dogmatic constants.

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* Alden Foster "Theoretical Frequency Curves" Transactions Am. Soc. C.E. Vol. 87 (1924) p. 148, states " .... runoff cannot have a value less than zero, "......"

J. J. Slade's partly bounded and wholly bounded functions in "An Asymmetric Probability Function" Proceedings Am. Soc. C. E. Vol. 60 (Oct. 1934) No. 8, Part I.

S Y N O P S I S

S E C T I O N I

EXCESSIVE PRECIPITATION ON SMALL DRAINAGE BASINS

In this section, only precipitations recorded in individual rainfall stations are treated. The method as used by the Miami Conservancy District in this connection involves the studies of depth and duration directly from the rainfall itself. The method proposed by the author, however, treats the total depth of rainfall from the consideration of storms producing such depths.

I - SCOPE OF INVESTIGATION

The article gives the fields of engineering to which studies of this section can be applied or have possible application.

II.- Purpose of Studying Frequency.

The article explains the necessity of frequency study from the consideration of its being a function of magnitude, also gives the practical purposes of such a study.

III - STUDIES BY THE MIAMI CONSERVANCY DISTRICT.

A brief outline of the method is introduced for subsequent criticisms.

IV - AUTHOR'S DISAGREEMENT TO THE MIAMI ENGINEER'S METHOD.

The author shows two points of fallacy in the Miami Engineer's Method in determining the frequency of excessive precipitation, with a conclusion that results from this method are not probable to say nothing of being true.

V - AUTHOR'S METHOD OF DETERMINING THE FREQUENCY AND MAGNITUDE OF EXCESSIVE PRECIPITATION ON SMALL DRAINAGE BASINS.

In this article, a method proposed by the author is introduced. The detailed descriptions and his opinion on the general statistical means and applications to engineering field are given in Section IV, to which reference is made. The article also explains why the rainfall data collected by the Miami Conservance District cannot be utilized, and shows additional mistakes in the Miami Engineer's Methods.

VI - AN EXAMPLE - BY METHOD OF NON-STATISTICAL DURATION CURVE.

An example of quadrangle 9-E is worked out by the duration curve from the data of Miami Conservancy District. The method is designed particularly for utilizing the Miami data. It is better than the Miami method but not as good as the author's method used in Article V.

SECTION II

EXCESSIVE PRECIPITATION ON SMALL DRAINAGE BASINS

I - SCOPE OF INVESTIGATION.

In the fields of Hydraulic Engineering which deal with storage of water, the engineer is concerned primarily with the full yield of a watershed and the variation in the annual amounts of precipitation. In some other fields of engineering, the monthly distribution is of chief importance; and in others still it is the excessive precipitation which occurs during shorter intervals of time which may require investigation. In the latter class, the problems of flood flows and flood control are usually involved.

The subject of this section finds its application in small drainage basins. The elements involved hereby are only depth and duration in relation to their frequency of occurence. Another important element, area, has not been taken into consideraton . The rainfall data employed are mostly the maximum totals of heavy precipitations which have been recorded in stations in the areas concerned. As in almost all cases, these values of rainfall depth will not cover considerable areas, it is evident that the application of present work will be limited to small watersheds, say of not more than about ten square miles, for the designs in those fields of engineering works such as:

1. Sewer systems.
2. Bridge and culvert openings.
3. Dams.
4. Levees.
5. Channel improvements.

II - PURPOSE OF STUDYING FREQUENCY.

The word frequency when applied to meteorological phenomena of irregular occurrence is best defined as the number of times, within a selected period of years, that a particular phenomenon has taken place. Dividing the period by the number of such happenings the quotient obtained is the average length of time in years during which the phenomenon/has happened once. This average number of years is also, though less accurately, spoken of as the frequency of the phenomenon.

So far as these phenomena of floods are concerned, there is a tendency to look upon a great flood as unprecedented and as likely never to occur again. As a matter of fact, great floods at long intervals are as much a part of natural course of events as are smaller floods at more frequent intervals. It follows, therefore, that the purpose of studying the frequency of excessive precipitation is to two-fold. First, it is the purpose to know the magnitude of the greatest flood to be protected against, in regard to its period of occurrence, for the determination of the extent of engineering works. The longer the period of occurrence anticipated, the greater will be the maximum flood to be expected. Nevertheless, it will be seen, that the depth of precipitation increases very slowly with frequency of more than once in one hundred years. Hence the maximum values of rainfall already recorded during the past century will probably not be greatly exceeded in the future. Secondly, it is also our purpose to determine the probable number of times that a land area will be inundated by different magnitudes of floods in a given period of years. One of the aims is to compare the land values before and after the provision of protection works. The designs are often materially affected by the desire to secure flood protection with a minimum damage to the land above dams. Therefore, the ability to make accurate estimates of the probable frequencies of floods of different magnitudes is of great usefulness.

When the problem in hand is the protection of a city where great loss of life may be involved, and where the property damage may be very great, the frequency of flood may be more or less immaterial, for in the protection of such cities there is no middle ground; no protection is adequate that is not complete, for the flood of a thousand years may come tomorrow. There are other classes of flood protection, however, where possibly something less than complete protection is warranted, and it always serves a useful purpose to be able to form some idea as what may be expected of flood relief works when considering their usefulness over a long period of years. A knowledge as to the laws of frequency, if there are any such, would further permit a better idea as to ultimate maximums than would be possible without such knowledge.

III - STUDIES BY THE MIAMI CONSERVANCY DISTRICTS.

In order to determine a basis for the engineering design of the structures for the protection of the Miami River Valley from floods, the engineering staff of the Miami Conservancy District, under the direction of A.E. Morgan, Chief Engineer, collected data of all storms of record within the United States up to 1914 in which the precipitation was defined as follows:

1. Where the normal annual precipitation is 30 inches or more: (a) Any one-day rainfall at one station

amounting to 10 percent or more of the normal annual precipitation; or (b) a total rainfall at one station amounting to 15 percent or more of the normal annual precipitation, regardless of the number of days in the period of excessive precipitation, so long as the average rainfall for the period is not less than one inch in 24 hours.

3. Where the normal annual rainfall is less than 20 inches, a total rainfall of 4 inches or more, regardless of the length of the periods, so long as the average rainfall for the period is not less than 1 inch in 24 hours.

Of the 4,316 stations where records were searched, 1,262 were west of, and 3,054 east of, the 103rd meridian which passes about 110 miles east of Denver, Colorado. An average of about 10 records of excessive precipitation was found for each station. Since the rainfall conditions in the western part of the United States differs materially from those in the Eastern part, no study was given to the precipitation occurring west of the 103rd meridian nor were the records of the precipitation prior to 1870 considered in their final study, because of the small number of stations in in existence before that date. From 1870 to 1914, a total of 3,641 periods of excessive precipitation were found. Of these, 12,36 were recorded at one station only, 996 at two to five stations, and 409 at six or more stations.

In order to utilize all existing rainfall records that possessed any value even though they differed materially in length, a method was adopted which is explained as follows: Assume a number of rainfall stations, say five for convenience of illustration, located within an area possessing uniform rainfall characteristics. At Station A complete records have been kept for a period of 70 years; at Station B, for 40 years; at Station C, for 60 years; at Station D, for 80 years; and at Station E, for 50 years, the aggregate of the period of record being 300 years. Treating this aggregate as a single record for the area under consideration, by the definitions of frequency, then it was assumed that the highest rainfall intensity recorded in the entire period occurred with a probable frequency of once in 300 years. Likewise the second highest intensity has been equalled or exceeded on an average of once in 150 years, and the third highest rainfall intensity has been equalled or exceeded on an average of once in 100 years. The process is capable of indefinite expansion, being limited only by the amount of data on hand.

To illustrate with an actual case, take the quadrangle of the earth's surface bounded by the 39th and 41st parallels and the 83rd and 85th meridians, in which the Miami River Valley is located, and which for convenience of reference has been designated as 9-E. This quadrangle contains 28 rainfall stations with a total period of record of 713 years, no stations having less than 10 whole years of observations being included. To ascertain what 24-hour rainfall intensity has been equalled or exceeded, on an average, once in 100 years, at any point in this quadrangle, select from the combined records the seven greatest 24-hour intensities, and the least of these is the desired rainfall intensity. Arranged in order of magnitude the figures are as follows:

Greatest 1-day Precipitation Records in Quadrangle 9-E
Aggregate period of record - 713 years.

1. Newport Barracks, Ky. May 24-25, 1858	6.35	inches
2. Urbana, O. Sept. 18, 1866	6.20	"
3. Cincinnati, O. June 17-18, 1868	6.00	"
4. Urbaba, O. June 15, 1866	5.95	"
5. Bellefontaine, O. March 25, 1913	5.61	"
6. College Hill, O. June 18, 1875	5.50	"
7. North Lewisburg, O. Sept. 27-28, 1884	5.40	"
8. Newport Barracks, Ky. Aug. 14, 1850	5.40	"

From the foregoing table it appears that an intensity of 5.40 inches in one day has been equalled or exceeded and is therefore likely to be equalled or exceeded in future years, on an average of once in 100 years at any point in quadrangle 9-E. This figure 540 has been called the pluvial index for the quadrangle corresponding to an 100-year period and a 24-hour rainfall intensity. By a similar process, the pluvial index can be obtained for 2 days, 3 days, 4 days, 5 days, and 6 days of greatest precipitation in a 100-year period.

In this way a series of 24 maps called isopluvial charts, was compiled showing graphically the pluvial index for each of the 133 two-degree quadrangles east of the 103rd meridian.

IV. AUTHOR'S DISAGREEMENT TO THE MIAMI ENGINEER'S METHOD.

There are two points with which the author cannot agree in the method described above as used by the Miami Conservancy District.

1. The Conservancy engineers considered the aggregate years of records of all stations available in the quadrangle as the period of a single record for the same, and the highest rainfall intensity among all stations having this period as its probable frequency. For instance, in the illustration quoted above, the aggregate of the period of

the record for the five stations is 300 years. Treating this aggregate number as a single record for the area under consideration, they assumed that the highest rainfall intensity recorded in the entire period has occurred with a probable frequency of once in 300 years, the second highest intensity has been equalled or exceeded on an average of once in 150 years and etc. This, as seems to the author, cannot be laid as a reasonable assumption unless the period of years kept in an individual station is chronologically different from each other. But usually this is not so. Periods of records available often occurred in same years. If this highest intensity ever recorded happens in Station C of 60 year period, why is it not equally possible for it to happen in Station D of 80 year period during the 60 year period of Station C that is included chronologically in the 80 year period of Station D? Under such conditions all that we can say is that of the highest intensity ever recorded in the quadrangle, the frequency for any point in the same must be greater than the chronological period of records aggregated from all stations.

It is evident that as long as the geographical and topographical conditions are uniform within an area under consideration, a storm of any intensity has equal chance to occur at any point within the area. Unless under exceptional cases, any such change of an appreciable amount in a quadrangle of 2^{0} latitude and longitude is usually rare. Therefore, it is correct to assign the lowest limit of frequency to the chronologically aggregated period of records among all stations, while it is not justified to fix the probable value as equal to the aggregate periods that occur essentially in the same years.

2. As previously mentioned, the studies of the present investigation finds its application only in small drainage basins of not more than about 10 square miles. Naturally the data to be used can be taken from records of individual rainfall stations. Since rainfall records of sufficient length are not available in any single station, resort to records of other stations within a two degree quadrangle possessing uniform climatological conditions will be the only way of solution. The problem, however, confronts at the outset, the difficulties of how to make use of the data from many stations to arrive at conclusions for a single station. The results of excessive precipitation derived from the method of the Miami engineers are apt to denote the mean frequencies corresponding to their respective magnitudes that may

happen in a point in the quadrangle since the average from all points is the same. For instance, in the same case as quoted above, it is assumed that the highest intensity occurred in Station C of 60 year period of records, and now suppose that the second highest intensity has occurred in Station A of 70 years period and that the two stations together have 90 years of record in chronology; then it is readily seen that the probable frequency of the second highest intensity may be taken as 90 years or less so long as the two highest records did not come from a single cyclonic storm or thunderstorm. While by the Miami Engineer's Method, it is $\frac{300}{2} = 150$ years. The criterion upon which this difference lies demands not much explanation. In the former way the result shows the possible occurrence of the second highest intensity within a 90 year period in any point in the quadrangle. It indicates the frequency of the occurrence of such an intensity, while by the Miami method the result shows the probable average frequency for the stated intensity in a point in the quadrangle by taking the mean of all available points (i.e. stations) in the same. It is the mean frequency of such an intensity rainfall. To express them in mathematical terms, let F = probability(or frequency) of occurrence of a storm producing any given intensity.

p = probability (or frequency) of the passing of such a storm over any point (station) in the quadrangle. (considering equal chance in any point) Then from the former way of reasoning, the frequency will be = P; while from the Miami Engineer's Method, it will be = p x P.

From the standpoint of the aim of application of present work, (for small areas) and of the unit used for investigation, (i.e., one station) we are led to conclude that the Miami Engineer's Method would not yield appropiate results for which we are in search.

V. AUTHOR'S METHOD OF DETERMINING THE FREQUENCY AND MAGNITUDE OF EXCESSIVE PRECIPITATION ON SMALL DRAINAGE BASINS.

As mentioned in Art. IV, in attacking such a problem of determining the frequency and magnitude of excessive precipitation on small drainage areas, we should keep in mind that first, it is the frequency of storms producing rainfall records of excessive magnitudes that we are in search of; and secondly, that such storms referred to are those which occurred in areas possessing the same geographical conditions. A method proposed by the author is embodied in the following processes:

1. Fixing the boundary lines of a geographical zone. (See Section III) From climatological studies, it is possible to determine such boundary lines of areas within

which the same meteorological conditions might be expected to occur as those of the place or city or watershed referred to. Similarly, the same magnitude and frequency of a storm might be expected in any point within the zone. These zones should be extended as large as the similarity of climatological conditions permits. Each zone is of course, not necessarily of uniform size. The method as used by the Miami Conservancy District of dividing into two degree latitude and longitude quadrangles is not desirable. The several adjoining quadrangles may usually possess same climatological conditions while in one quadrangle the condition may be abruptly changed. We cannot, therefore, afford to study the frequency of storms and magnitude of rainfalls from the quadrangle divisions.

2. Collecting all rainfall data in stations within the same zone. These data should, of course, include the periods and dates of occurrence, the depths of rainfall recorded, and the locations and the length of records of the stations. Next group together those records of different stations that occurred during a same storm, the maximum of which, considered as the maximum depth of rainfall that can be expected from such a storm in passing the geographical zone concerned, is entered

on to another data sheet. The records thus picked out should not have any lower limit such as the Miami Engineers did in defining its excessive precipitation. The reason is explicit: As we are studying the statistical occurrences of temendous storms the records of extremely small storms bear the same importance as those of the big storms in studying their distributions. This is another reason why the data collected by the Miami Conservancy District cannot be used for such a study.

3. Applying the statistical method.

The author's opinions about theories and applications of the method are given in Section I ; and in Art. X of the same section, the best types of frequency curves available at present were mentioned. The general procedute is as follows: After arranging the rainfall records according to the order of their magnitudes, the mean value is figured out which will be taken as the vertical axis of the frequency curve. Then take the second, third, and fourth moments of all records expressed as ratios to the mean to get μ_2, μ_3 and μ_4 respectively. Next calculate

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \qquad \beta_2 = \frac{\mu_4}{\mu_2^2},$$

and

$$K = \frac{\beta_1 \quad (\beta_2 + 3)^2}{4\,(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$$

The value of K will determine to which of the seven Pearsonian types the set of data belongs. For types I and III, H. Alden Foster has simplified the process by providing ready made duration curves.* For other types, frequency curves should be integrated either algebraically or graphically in order to get duration curves, from which the percentages of happening can be picked out for any depth of rainfall.

In case where the Pearsonian Types cannot give good fittings to the data, the approximate Gram-Charlier Series is suggested to be the next best available. The same values of μ_2 and μ_3 as calculated and the coefficient of variation σ and coefficient of skewness K are used. The author has prepared a set of duration curves for different coefficients of skewness. (Art. IX, Section I) The depth of rainfall corresponding to any percentage of happening can readily be interpolated.

4. Determining the corresponding values of frequency and magnitude. The percentage of happening of storms greater than the corresponding depth of rainfall as determined from the above, represents the number of storms that might be expected in every hundred storms in

in a very great number of storms. This does not, however, show the frequency in years of happening at all. Nevertheless, we can find the normal interval in days of succeeding storms either from meteorological studies (the U.S. Weather Bureau provides this) or directly from the set of data just referred to. The average interval equals the chronological period of records for all stations in the zone (transformed into number of days) divided by the total number of storms in this period. The normal or average interval so found divided by any percentage of happening will give the frequency in days of happening of storms greater than the corresponding depth of rainfall to the percentage referred.

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* Trans. Am. Soc. C.E. Vol. 87, (1924) Theoretical Frequency Curves.

VI. THE METHOD OF ARBITRARILY ADJUSTED DURATION CURVE WITH AN ILLUSTRATIVE EXAMPLE

As stated in Art V, on account of absence of the rainfall data of depths below the limits of excessive precipitation that the Miami Conservancy District defines, those data above the limits cannot be used for estimating extreme values in applying the statistical method. Nevertheless, in order to utilize the present data available, another means is hereby introduced - The method of arbitrarily adjusted duration curves. An example worked out below might well explain it.

Take the quadrangle 9-E bounded by the 39th and 41st parallels and the 83rd and 85th meridians, in which the Miami River Valley is located. This quadrangle contains 75 rainfall stations (total number in history) some of which recording from the year 1850; and up to 1931, the aggregated chronological period is 82 years. The "Excessive Precipitation Sheets" prepared by the Miami Conservancy District for the quadrangle are collected and storms rearranged according to the order of magnitude of maximum 1-day precipitation as shown in the table. The column of "Average number of storms per year above the ppt." is found by dividing the quantities in the column of "number of storms above the ppt." by the total period of record of 82 years. These are then plotted in the logarithmic probability paper in order to reduce the curvature of the curve due to irregularities. A straight line drawn near the majority of the joints is an "arbitrarily adjusted duration curve."

That part of the data below 2.55 inches are not used in plotting. The reason is that the definition of the excessive precipitation does not give a fixed lower limit for all stations so that in the range of this part of data from 2.55" to 1.15 "available in the "Excessive Precipitation Sheets" it will probably contain other quantities from some stations that are discarded due to the relatively higher limits of the same.

It should be kept in mind that the present method is simply one of fitting the data. It is non-statistical. Nevertheless, it possesses a much more sound logic than the method used by the Miami Conservancy engineers on account of its avoidance of the two fallacies of the latter as mentioned in Art. IV.

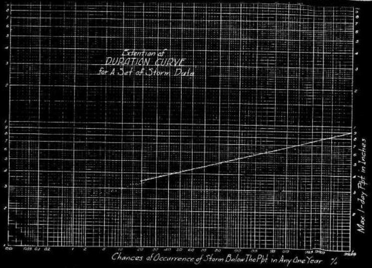
The result from the Miami Engineer's Method as given in Art. III is 5.40 (so called "Pluvial index) inches for quadrangle 9-E corresponding to a 100-year period and a 24-hour rainfall. The quantity is 6.68 inches from the present method, showing quite a great discrepency.

For a 1000-year period, the maximum one-day precipitation is 7.8 inches, while the Miami Engineers Method cannot show any result whenever the period is over the aggregated period of records(for quadrangle 9-E, the period is 713 years). This is remarkably another merit of the present method.

Greatest 1-day Ppt. Records in Quadrangle 9-E
Chronologically Aggregated Period of
Record = 82 years.

Year	Storm Period Month	Storm Period Day	Station where the greatest ppt. of the storm recorded	No. of storms above the ppt.	Average No. of storms per yr. above ppt.
1873	6-7	26-4	Carthagena, O.	0.5	0.0061
1858	5	24-25	Newport Barracks, Ky.	1.5	0.0183
1866	9	18	Urbana, O.	2.5	0.0305
1868	6	17-18	Cincinnati, O.	3.5	0.0427
1917	9	7-9	Fernbank, O.	4.5	0.0549
1913	3	25	Bellefontaine, O.	5.5	0.0671
1875	8	18	College Hill, O.	6.5	0.0793
1899	8	4-5	New Paris, O.	7.5	0.0915
1884	9	27-28	North Lewisburg, O.	8.5	0.1037
1850	8	14	New Barracks, Ky.	9.5	0.1159
1905	6	4-7	North Lewisburg, O.	10.5	0.1281
1877	6	15-25	Carthagena, O.	11.5	0.1403
1897	3	1-5	Camp Denison, O.	12.5	0.1525
1876	1	23-28	Kenton, O.	13.5	0.1647
1880	8	17-20	Bellefontaine, O.	14.5	0.1769
1896	8	6-7	Richmond, Ind.	15.5	0.1891
1931	7	17-22	Tippecanoe City, O.	16.5	0.2013
1925	9	13-18	Germantown, O.	17.5	0.2135
1876	7	10-14	Kenton, O.	18.5	0.2257
1877	3	9-16	Kenton, O.	19.5	0.2379
1886	5	10-15	Dayton, O.	20.5	0.2501
1865	4	9-11	Albania, O.	21.5	0.2623
1922	6-7	27-3	Hilkboro, O.	22.5	0.2745
1907	3	9-14	Cincinnati, O.	23.5	0.2867
1934	6	5-10	Grumilk, O.	24.5	0.2989
1926	9	1-5	Richmond, O.	25.5	0.3111
1920	8	20	Hamilton, O.	26.5	0.3233
1905	8	11-15	Delaware, O.	27.5	0.3355
1907	7	16-20	Bellefontaine, O.	28.5	0.3477
1871	5	30	Cincinnati, O.	29.5	0.3599
1893	8	4	Springboro, O.	30.5	0.3721
1924	6	8	Pleasant Hill, O.	31.5	0.3843
1910	10	5-6	Waynesville, O.	32.5	0.3965
1898	7	20-24	Pattsburg, O.	33.5	0.4087
1875	8	1-5	Carthagena, O.	34.5	0.4209
1908	6	20-21	Grumilk, O.	35.5	0.4331
1926	7	27	Waynerville, O.	36.5	0.4453
1871	8	25-29	Kenton, O.	37.5	0.4575
1876	2	23-28	Kenton, O.	38.5	0.4697

2.34	1903	6	5-10	Dayton, O.	
2.34	1937	11-12	27-1	Tippecanoe City, Ohio	
2.20	1898	9	24	North Lewisburg, Ohio.	
2.20	1926	8	16-32	Wilmington, O.	Not used
2.15	1900	5	18	Camp Denison, O.	
2.01	1851	11	9-14	Newport Barracks Ohio	in
1.91	1906	8	20	Camp Denison, O.	Plotting
1.82	1851	12	26-31	Newport Barracks Ohio	
1.46	1899	7	25	Camp Denison, O.	
1.15	1889	7	14	Dayton, O.	


Extention of
DURATION CURVE
for A Set of Storm Data
Chances of Occurrence of Storm Below The Ppt in Any One Year %
Max 1-day Ppt in Inches

SYNOPSIS

S E C T I O N III

EXCESSIVE PRECIPITATION ON LARGE DRAINAGE BASINS

This section is devoted to a general description of climatic controls on the rainfall characteristics in causing floods in regions in any part of the world; and to a detailed description of the storm control particularly in Eastern United States. Some general analysis of flood flows and a method of estimation from storms are herein presented. The data used are mainly from those collected by the Miami Conservancy District contained in Technical Reports Part V.

I - THE GENERAL CLIMATIC CONTROLS. This is an enumeration of the general controlling factors which combine to produce any given climate.

II - ANALYSES OF FLOOD FLOWS. This treats some attempted analyses of flood problems in three representative types of climate in the temperate zones of the world. A general scheme to attack the problem particularly in Eastern Asia or China proper as suggested by the author is most distinctive.

III - SCOPE OF INVESTIGATION. The aim and characteristics of the present study are pointed out.

IV. Formation of Cyclonic Storms. This is a general description of cyclonic storms, especially of those happening in the Eastern United States from the meteorological point of view.

V. Miami Conservancy District studies on the Effect of Great Storms. This is a brief introduction of their studies.

VI. Geological Location and Seasonal Distribution of Great Storms. Some discussions in regard to these points are given from studies of the results of the Miami Conservance District.

VII. Discussions of Time, Area, Depth Relations. This introduces some discussions quoted from the District report with explanations by the author.

VIII. The Proposed Division of Violent Storms Zones in the Eastern United States. It describes the purpose, basis, and methods upon which the author proposes the division of storm zones in the Eastern United States.

IX. Standard Maximum Time-Area-Depth Curves in the Zones of the Eastern United States. Five sharts containing the time-area-depth curves for each zone are presented.

X. Use of the Time-Area-Depth Curves. It describes the method of using the data in a practical manner.

XI. The Proposed Method of Extreme Flooding. This is another method proposed from the considerations of the amount of water brought over by great storms. Statistical estimation is here introduced.

SECTION III

EXCESSIVE PRECIPITATION ON LARGE DRAINAGE BASINS.

I. THE GENERAL CLIMATIC CONTROLS *

Climate is briefly defined as average weather. Means or averages may, however, be made up of widely differeng values of the elements which go into them. Therefore a satisfactory presentation of climate must include more than mere averages. It should also take account of regular and irregular daily, monthly and annual changes, and of the departure, means and extreme from the average conditions which may be expected to occur at the same place in the course of time. So climate is the resultant of many variables. One climate differs from another because of a different combination of the controls.

The general controlling factors which combine to produce any given climate may be enumerated as follows in order of their comparative importance.

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* Based Generally on Ward's "The Climate of the United States" whth some changes made.

(1) Latitude or Solar Control. The sun is obviously the fundamental control of climate. The general distribution of temperature over the earth's surface, as well as the diurnal and seasonal changes, depend upon variations in the intensity and in the duration of sunshine. If the sun alone were concerned, all places of the same lattitude circle would have the same climate, for the intensity and amount of sunshine depend upon the angle of incidence of the Sun's rays and upon the length of day, and both of these can be accurately determined from astronomical calculation for any latitude. Such a condition is very decidedly modified by the distribution and influence of the succeeding controls.

(2) Land and Water. The influence of latitude may be wholly overcome by the effects of land and water. Land and water are fundamentally different in their behaviour regarding absorption and radiation. Land areas and the air over them warm and cool readily and to a considerable degree; water areas and the air over them warm and cool slowly and relatively little. This radical difference in the manner of warming and

cooling between land and water areas causes the changes of climate in two ways: First, the seasonal changes of general wind direction over larger continental area upset all climatic conditions by the so-called monsoons produced in this way. In India and Eastern Asia, this effect is so predominant as to deserve the name of "Monsoon Belts". The summer monsoon rainfall results from the inflow of a large body of warm, moist air from the sea on to the land area; a consequent retardation of the velocity of air currents, as the result of friction, and an ascent of the air, the rainfall being particularly heavy where the winds have to climb over high lands. Thus, in India the precipitation is heaviest at the Bay of Bengal, where Cherrapunzi, at the height of 4455 feet in the Khasi Hills, has a mean annual rainfall of between 400 and 500 inches. Secondly, for those parts of continents near the o[illegible] side of oceans, being exposed to the influence of the ocean, with the prevailing winds blowing directly from the conservative ocean, the climates are on the whole relatively mild and equable, with slight seasonal fluctuations - known as "Marine Climates". This is true in the western Europe and the Pacific Coast of the United States.

(3) Mountain Barriers. Mountain ranges especially when high and extended, are effective climate barriers. If they stand in the path of the prevailing winds, they may bring about marked differences in rainfall, in temperature, in cloudiness, in humidity, on their opposite sides. When near a coast, especially a windward coast, they prevent ocean influences from extending inland. Thus the Pacific coast ranges – Cascades, Sierra Nevada, Coast – prevent the influence of the Pacific from being carried far into the continent and thus separates a narrow coastal belt, much of which has a modified marine climate, from an interior, east of the Sierra Nevada-Cascades, where the rainfall is less and the ranges of temperature are much greater. In China, the Mount Ying Ranges (陰山脈) however, are not high enough as a barrier to prevent the winter monsoons blowing southeastward and carrying fine sands from the Great Mongolia Desert from being carried far into the Central China as to give the troublesome loess deposits. The author often dreams of the change of everything in the central part of his country if the Mount Ying Ranges were elevated 3000 feet higher in another geologic epoch.

(4) Prevailing Winds. The prevailing winds are, in fact, a partial result of the latitude of the region. In belts between latitude 30° and 60° north it is chiefly the south-westerlies that prevail. In belts between 25° north and the equator the n orth-eastern trade winds prevail. As winds are of critical importance in controlling weather types, their direction and velocity must be considered in any study of climate. The prevailing wind in summer may be a very warm one, as is the case over most of the Eastern United States, where south-westerly wind directions are dominant during the hot months. Such conditions naturally increase the summer heat. Or the prevailing winter wind may be a cold one, as in New England, thus making the winters more severe. The great permanent areas of high and of low pressure adjacent to a continent - the so-called "Centers of Action" also play a considerable part in determining the directions of the prevailing winds on the continent.

(5). Altitude. In addition to the barrier effect, mountains and highland have certain special climatic peculiarities because of their elevations above sea level. It is here that the control of climate by altitude is met. Mountain and plateau climates are always placed in a group by themselves as distinguished from those of lowlands. The former as contrasted with the latter, are characterized by a general decrease in pressure, temperature and absolute humidity; and an increased intensity of insolation and radiation; higher wind velocities; usually a greater frequency of rain and snow, and, up to a certain altitude, more of it. Again, the local topography is of prime importance in bringing about modifications in climatic conditions, as for instance mountains both modify the general winds and give rise to local winds.

(6) Storm Control. As climate itself is the resultant of diverse weather conditions, cyclones and anticyclones which determine the weather from day to day are essential climatic controls. If the weather of a certain section happens to be distinctly under storm control for part of the year, like the Eastern United States during winter, the influence of the other controls, such as latitude, land and water, altitude, etc., may be largely or wholly overcome by the conditions resulting from the prevalance of cyclonic winds and clouds.

this subject is of sufficient importance to warrant its consideration in latter articles.

(7) Ocean Currents. The ocean currents can have important influence on the climate of an adjacent land only when the wind is blowing on shore. Further, ocean waters themselves, with the help of any ocean currents, are conservative bodies, and therefore tend to temper the cooling or the heating of any land over which their influence may be carried. It is evident that the Gulf Stream and the Gulf Stream Drift do keep the North Atlantic waters off the eastern coast of the United States warmer than they would otherwise be, and that the Laborador Current is a cold flow which chills these same waters to a lower temperature than they would otherwise have. And on the Pacific side the Japan Current, flowing southward along the coast with a subordinate eddy circulating around the Gulf of Alaska, certainly helps to keep the Pacific slope climates of America rainier and more temperate than they would be without this current. A glance at the isothermal charts of the world at once shows the effect of these currents in deflecting the isotherms.

II. ANALYSES OF FLOOD FLOWS.

The flooding of a basin along the banks of a river is either the direct or indirect result of excessive precipitation on it. The study of agents causing such excessive precipitation is necessarily associated with the study of floods. These agents should be searched from the type of climate resulting from the combination of the climatic controls as described in the above article. Let us treat only the flood problems in regions of temperate zones for our interest.

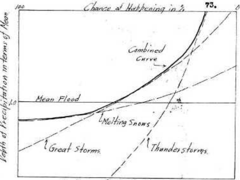
<u>The United States of America east of the Rockies</u>. The continental area of the United States lies almost entirely within what is generally known as the belt of prevailing westerly winds. These are members of the general atmospheric circulation. The local influences of the changing seasonal pressures over the continent and over the adjacent oceans, however, are to a large extent paramount to the general control over such constant air movement. The general configuration of the country, the trend of mountains and of valleys; locations to windward or to leeward of mountains or of lakes; exposure to land and sea breezes - all these

factors have a part in controlling the climate. Then, varying from day to day, more temporary than any of these other controls, comes the ever-changing influence of cyclones and anti-cyclones. The dominance of these passing conditions over air movements is often so complete that easterly winds are of frequent occurrence throughout the belt of the prevailing westerlies. There is thus a great ring of stormy weather over the temperate zones as a whole, oscillating poleward and equatorward as the sun moves to and fro in the course of its regular migration. Northward across the United States swings this storm belt as the summer sun comes north of the equator. Southward it swings in winter, following the declining sun, covering the country even to the Gulf of Mexico. Scattered through the southern quadrants of the larger cyclonic storms, especially during the warmer months, come the local disturbances - thunderstorms and tornadoes. This second belt of local storms south of the general cyclonic storm belt also swings back and forth seasonally, covering practically the whole country in summer and being carried well into and even across the southern states in winter. Therefore, the weather types of Eastern United States varying seasonally and geographically, result from a combination of periodic diurnal elements, under the control of the sun, (the local storm control) and of non-periodic, cyclonic and anti-cyclonic elements.(the cyclonic storm control) the former is predominant in summer and the latter in winter.

The above discussion of the climates of Eastern United States leads us to conclude that the causes of all floods in the States may be classified as follows:

(1) Great storms of wide distribution.
(2) Thunderstorms covering only small areas.
(3) Melting snows.

It is true, of course, that these causes grade into each other and no line of demarkation can be drawn, but for the purpose of explanation, let us consider them separately. Each cause can be represented by the curve platted coordinates of depth of precipitation and percentage of chance of happening as given in the following diagram:



Analysis of Flood Flows For The U.S.A.

The floods from the melting snows will be the most regular and will have the smallest coefficient of variation because the rate of melting depends upon the rise in temperature at a certain season the year and temperature conditions are in a general way less variable than rain conditions. So the curve has a general smaller inclination. The floods resulting from great storms will vary much more in size and will therefore be represented by a steeper curve on the plotting. Floods due to thunderstorms may happen once in 1000 years, but when such a flood does come it will be big. The curve representing such floods will therefore have steepest inclination.

It is apparent that each curve will control where it is above the other two; and that the curve showing all of the annual floods will be made up of parts of the three curves, each part being that curve which is above the other two.

Melting snows constitute a flood factor mainly in the north and in streams flowing from high mountains in all parts of the United States where the climate is warm, this curve can be omitted from the diagram.

EASTERN ASIA OR CHINA PROPER

The Continental area of the Republic of China also lies mainly in the belt of the prevailing westerlies. Cyclones and anti-cyclones sweep over the country one after another the same as in the United States of America, but less in strength. Here the general seasonal pressure control over the winds due to the effect of land water is more emphatic and the storm control is relatively less important, so that the northwesterly winds remain in much less disturbed possession. The latter are indeed so marked that they deserve the name of winter monsoons. For this reason, China has drier and less cloudy winters than those which characterize the Atlantic seaboard of North America.

In June and July, the weather is wet, sultry and oppressive. Now a large body of warm, moist air flowing from the sea on to the land area results in summer monsoon rainfall. This happens each year almost always at the time when plums turn yellow and so it is named the "Yellow Plum Raining" by the Chinese(黃梅雨).

The raining usually has a continuous period of as long as three weeks with some intermissions at times of one or two days. It is this summer monsoon rainfall that makes southern China particularly suitable for raising rice which constitutes the main ration of the Southern Chinese. It is, however, this same summer monsoon rainfall, accompanied by the tremendous depth of loess sand brought over the Central China by the winter monsoon that makes the people living there suffer from the disastrous inundations and the resulting famine. It is due to these monsoons that the hydraulic engineers of the world find difficulties in planning the flood control of the Yellow River.

In devising flood control problems of rivers in the United States, a seven day duration rainfall would usually be considered as rather sufficient for determining the maximum depths over the catchment basins. For the same problem in China, this duration of rainfall is far from enough. A three week period rainfall is generally not uncommon in any part of the China proper, and designs should be made to provide this. It is rather ridiculous that some famous foreign engineers who were invited to China, designed the river works by providing the flood capacities calculated from the Weston E. Fuller's Formulas. This wide difference in duration of rainfall between China where the major control is the monsoons and the United States where the storm control predominates, makes no single formula as commonly used in the latter applicable to the former. This idea, never noticed by anybody so far as recorded, should be particularly borne in mind for those who are engaged in the river works of China.

It follows, therefore, that the causes of floods in China or Eastern Asia may be classified as follows:

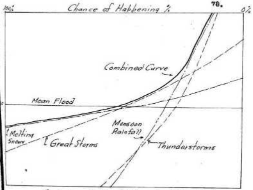
(1) Summer monsoon rainfalls of practically complete covering over the whole drainage basin.

(2) Great storms of wide distribution.

(3) Thunderstorms covering only small areas.

(4) Melting snows.

These factors can be represented in a similar way as above described by the following curves:



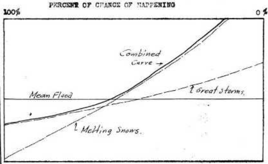
Analysis of Flood Flows in China Proper.

Here, on account of the effect of summer monsoons, the combined curve will give more depth of precipitation for rare chances of happening than in the case of Eastern United States.

No attempt, of course, is made to give numerical values to the curves in either diagram because such values are obviously beyond reach under present conditions.

WESTERN EUROPE AND PACIFIC COAST OF THE UNITED STATES

In these regions, marine climatic types are carried by the prevailing westerlies on to the western coasts of the continents, giving them mild winters and cool summers, abundant rainfall and a high degree of cloudiness and relative humidity. North-western Europe is particularly favored because of the remarkable high temperatures of the North Atlantic Ocean, and because of the influence of the winds controlled by the low pressure area off Iceland. Along the western coast of North America and of Europe the mean annual ranges of temperature are under 25 degrees. The rainfall is yet well distributed throughout the year, with the marked maximum in the fall and winter which is characteistic of the marine regime. This makes the problem of flood control much easier if other conditions are equal. However, flood flows usually result from melting snows that are accumulated in higher altitudes, and which element now becomes of major control.



Analysis of Flood Flows For Western Entope

III. SCOPE OF INVESTIGATION.

As aforementioned in the last article, the principal factors causing the flooding of a river in any part of the world are cyclonic storms, thunderstorms, monsoon rainfalls and melting snows. These causes grade into each other and no lines of demarkation can be drawn. There are not enough data with respect to floods from thunderstorms to serve at the present time as a basis for any rational method of study. On melting snows many studies have been made, and since they are more regular, the results can be expected of high reliability. Again, they constitute a flood factor mainly in the north and in streams flowing from high mountains and are of no importance for other regions than that mentioned. The monsoon rainfalls are the prime control for flood flows in Eastern Asia, but there are only small amount of data available for any intensified research. We, therefore, had better limit our investigations to those of the cyclonic storms. It is the major climatic control as well as that of the excessive precipitation over the United States east of the Rockies and is especially emphatic during winter and spring seasons.

The discussion of rainfall statistics in Section II was limited to a consideration of excessive precipitation records at individual stations. This involved the two important rainfall factors of depth and duration, but ignored a third factor, area, which is equally important. The next logical step in the investigation, therefore, is to study a number of large storms as a whole, giving consideration to all three of the factors, time, area, and depth, which determine the size of storms.

It is natural that rainfall conditions should be studied in an effort to understand and account for flood flows and to get an idea of the probability of the recurrence of the conditions that have produced destructive floods.

The United States Weather Bureau has a tremendous amount of data in regard to rainfall at a large number of stations and extending over many years. In flood-flow records there is nothing approaching this volume of data even in a remote degree. If definite relations between rainfall and run-off could be established, the problem would be simplified. Plenty of hard work has been done upon the study of these relations, and much has been learned. The relations are variable because there are innumerable other factors, beside rainfall conditions that must be taken into account since they affect the proportions and the rapidity of run-off.

IV. FORMATION OF CYCLONIC STORMS

A cyclonic storm occurs either in the temperate latitudes or begins with in the tropics and subsequently changes when it enters the temperate. These storms are depicted on the daily weather maps as areas of low pressure, and appear in great numbers and in almost infinite variety as regards position and form. The ceaseless changes in the weather of the United States are due almost entirely to the approach and passage of these areas of low pressure. For this reason, they are sometimes spoken of as the lows of the weather map, or simply "lows". In these areas, spirally inflowing winds turn counterclockwise in the north hemisphere and clockwise in the southern. The wind velocity is generally moderate; the accompanying cloud area is immense; precipitation usually occurs; the changes in temperatyre and humidity are large and well marked. The whole formation is from a few hundred to several thousand miles in diameter and moves with moderate velocity from some westerly to some easterly quarter.

If an observer far above the earth could look down upon the whole northern hemisphere, he would see a ceaseless procession of lows between the thirtieth and eightieth parallels of latitude, moving eastward and encircling the poles. Then big cloud areas would gleam white in the reflected sunlight. The reason for this eastward motion is because this is the prevailing direction of both the surface winds and the fast-moving upper air currents in these latitudes. The lows thus drift with the general wind system.

Lows may originate anywhere. A good part of those which visit Europe originate over the Atlantic Ocean. In the United States, a favorite place of origin is the Canadian Prairie Provinces just east of the Rocky Mountains. Some of them come through the mountains from the north Pacific, while a few originate over the Zarragossa. The length of the path followed varies from a few hundred miles to, in a few rare cases, more than half of the circumference of the globe.

The tropical cyclones come up from the tropics and join the extratropical ones in two places: Over the West Indies and over the Philippins and Japan. As soon as the tropical cyclone enters the extratropical region it loses its violence.

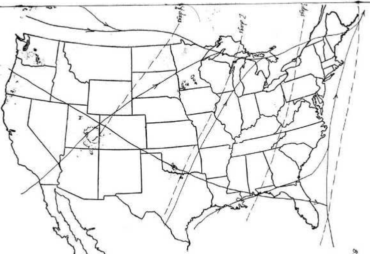
If the smaller portion of the world is considered, as for example Europe, it is found that not all areas are covered by the same number of lows. The lows seem to prefer to travel certain rather definite paths; and if the actual paths followed by the lows for a number of years are generalized or summarized, it is made evident that a more or less definite system of storm tracks prevail.

It is said by Willis I. Milham in his "Meteorology" that "It is probably safe to affirm that no subject in meteorology has been more thoroughly studied by the statistical method, that is, by generalizing statistics, than have the highs and lows. Since the files of daily weather maps, particularly for the United States and Europe, cover a period of from thirty to forty years, the material for such a study is ample. All the statements which have been made concerning the distribution of the meteorological elements about highs and lows, their tracks and velocity of motion, have been gained by the inductive method of reasoning and are based upon the general conclusions which have been drawn from tables of statistics about highs and lows".

One disadvantage of the storm statistics, however, is that meteorologists have not so far given any complete explanation to the origin of lows. This makes the branch of science not well established. The present theories: origin of eddy currents, lows resulting from highs, and Bigelow's Counter current theory are not satisfactorily and well acknowledged. No attempt will be made to explain either of them.

The tracks of lows across the United States have been generalized by Bigelow, Russell and VanCleef. The Bigelow system is the most highly generalized so that there will be only small percentage of lows which will follow these tracks. The Russell system is the next, and the VanCleef system is the least generalized of the three, thus accounting for the paths followed by the largest number of lows. As a low may loiter in one place for a day or two, or turn sharply aside in one direction or another or more erratically in almost any direction, these generalized tracks simply respresent normal behaviour.

In the figure following the Bigelow system of tracks is shown. The main track follows the northern boundary of the United States across the Great Lakes and out the St. Lawrence Valley. This main track is joined by three others coming up from the south. One comes up from the Colorado and Utah and joins it near Lake Superior. Another comes up from Texas and joins it near Lake Huron. The third comes up the Atlantic Coast and joins it near Nova Scotia. There is a second rain track across the Texas and the Gulf States to the Atlantic coast, where it either turns northward or goes out over the ocean. The broken line shows the average daily movement.



The Bigelow System of Storm Tracks across The United States
(U.S. Weather Bureau)

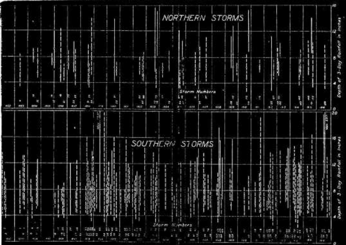
V. MIAMI CONSERVANCY DISTRICT STUDIES ON THE EFFECT OF GREAT STORMS.

After the Dayton flood of 1913, the Miami Conservancy District was formed to build reservoirs, to enlarge river channels and to do other work necessary to prevent recurrence of flood damage. Arthur E. Morgan, Chief engineer of this District, carried out a study of the Weather Bureau records that is unique. The results are obtained in Part V of the Miami reports. The study covered all records of the United States Weather Bureau east of the Rocky Mountains, and the results have been arranged so that it is possible to see what these records indicate for high rates of precipitation and so that they are of practical use when applied to actual problems.

Preceding the year 1892, the number of rainfall observing stations was too few to furnish sufficient records to determine with precision the areas covered by important storms. Since that date the rainfall stations have been relatively more numerous, constant, and uniformly distributed. Hence for the purpose of studying frequency, seasonal and geographical distribution of large storms, the 35-year period, 1892-1916, was adopted.

In this study all storms were included that had records at five stations of 3-day precipitation equalling or exceeding six inches. Records of 160 such storms were found. They were divided into two groups, 47 northern storms and 113 southern storms, together with three other important previously occurring storms. The line of division between north and south was somewhat arbitrarily chosen along the north boundaries of North Carolina, Tennessee, Arkansas, and Oklahoma. This grouping of the storms has certain conspicuous defects, as it places Iowa and Illinois storms in the same class with thos of New England, although the sources and characteristics of the two types are quite different. Nevertheless, the subdivision has proved to be of material assistance not only in making an intelligent analysis possible of the relative sizes of the storms, but also by bringing out clearly facts relating to their seasonal distribution.

The relative sizes of the storms were determined by showing graphically the twenty highest 3-day rainfall records in each storm. To do this each storm was given a separate ordinate on a sheet of cross-section paper, and on this ordinate were platted the highest,



CHRONOLOGICAL DISTRIBUTION OF 47 GREAT NORTHERN STORMS AND 113 GREAT SOUTHERN STORMS DURING 25-YEAR PERIOD, 1892-1916.

[illegible] range of the records at the 20 stations reporting greatest rainfall for the maximum 3 days is shown by a vertical line for each storm. The upper and lower [illegible] mark, respectively, the fifth and tenth greatest values; and the upper and lower extremities of the lines mark, respectively, the highest and twentieth highest [illegible] [illegible] which maps and time-area-depth curves were drawn. The storms can be identified by means of the numbers near the lower margin.

the fifth highest, the tenth highest, and the twentieth highest values for the maximum 3-day period of the storm. The accompanying figure plotted in this manner, shows the 47 great northern storms and the 113 great southern storms arranged in chronological order. The upper and lower extremities of the lines representing the storms show, respectively, the highest and the twentieth highest values; the upper and lower circles show, respectively, the fifth highest and tenth highest values. The solid lines represent storms for which maps and time-are-depth curves were drawn.

The 3-day maximum period was chosen for several reasons. Many intense storms last only 1, 2, or 3 days, and obviously it would not be wise to compare the maximum records of such short storms with records representing the total precipitation of 5-day or 6-day storms. On the other hand, it would not be fair to compare the maximum 1-day records of a 6-day storm with the maximum 1-day records of a 1-day storm. The 3-day period is therefore, considered a fair average for the purpose of comparing these types of storm.

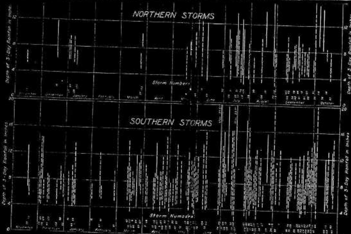
The report then proceeds with a description of some thirty-three of the most notable storms that have swept the country in the entire period covered by the Weather Bureau records. The work consists of a number of distinct steps as follows: (1) The rainfall data pertaining to the storm is assembled. (2) There are determined the 1-day period of greatest average rainfall, the 2-day period of greatest average rainfall, and so on, until the whole duration of the storm is covered. (3) On a large scale map of the United States, showing all rainfall observation stations, there is plotted at each station the figures showing the amount of precipitation for the maximum 1-day of the storm. Similar maps are prepared for the successive periods until the whole duration of the storm, or until the maximum 5-day period is covered. For storms of more than 5-days the mapping was confined to the five consecutive days of maximum rainfall. (4) On each map isohyetalsare drawn. To cover all the days of the storms used for this study, 114 such maps were required. These maps have been reduced to a small scale for publication omitting the individual records. (5) The areas contained within the isohyetals are measured with a planimeter. (6) The average depths of rainfall over the areas are calculated. (7) On coordinate paper a curve, designated a "time-area-depth" curve, is platted using the results obtained from each map, platting as ordinates the area in square miles contained within each isohyetal and the average depth in inches over such area.

The results thus derived are expected to apply to different sections of the Eastern United States, in determining the most probable distribution and intensity of great storms which may occur in the future.

VI. GEOGRAPHICAL LOCATION AND SEASONAL DISTRIBUTION OF GREAT STORMS.

The general geographical and seasonal distribution of the 160 great storms studied by the Miami Conservancy District are shown in the following chart. From a study of the chart, it was decided by the District engineers to divide the year into quarters beginning respectively,- November 1, February 1, May 1 and August 1. In the first half of the year, during the months of November to April, all storms, as above defined, occur in the Mississippi Valley from the Gulf of Mexico to Iowa, Illinois, Indiana and Ohio. In the second half of the year, May to October, they occur principally along the Atlantic and Gulf Coastal regions and in the Central Mississippi Valley, the great number of interior storms occurring west of the Mississippi River. In both north and south the greatest storms occur during the summer months. In the north, they tend to occur frequently in late summer; and in the south, most frequently in the early summer. Throughout the entire year storms are much more numerous in the south than they are in the north.

During the 25-year period in question, 46 storms



THE 47 GREAT NORTHERN STORMS AND 113 GREAT SOUTHERN STORMS OF THE 25-YEAR PERIOD, 1892-1916, ARRANGED BY SEASON OF OCCURRENCE.

The range of the records at the 20 stations reporting greatest rainfall for the maximum period of 3 days is shown by a vertical line for each storm. The upper and lower [illegible] in each line mark, respectively, the fifth highest and tenth highest values; and the upper and lower extremities of the lines mark, respectively, the highest and twentieth highest values. The solid lines represent storms for which maps and time-area-depth curves were drawn. The storms can be identified by means of the numbers near the [illegible] margin.

have occurred or approximately 3 storms each winter season. Of this number, 38 were in the southern group, quite evenly distributed over western Georgia, Alabama, Mississippi, Louisiana, Arkansas, and Eastern Texas. This means, of course, that a winter storm of the defined intensity and size occurs on an average of once every three years in the Upper Mississippi Valley, and once or twice each year over some part of the southern states. For summer storms, there were 112 in number over the areas along the Atlantic and Gulf coast and along the Mississippi River. This is an average of 4 or 5 each year somewhere in the United States east of the 103rd meridian. Of this number, 12 storms, or an average of 1 storm every 2 years, occurred on the north Atlantic coast, and 26 storms or an average of 1 each year in the interior region along the Mississippi River.

That some parts of the United States are apt to have more chance of passing storms or have storms yielding heavier precipitation than other parts are apt to have, may be attributed to some or all of the following climatic controls:

(1) The latitude of a place within the United

states is always the dominant factor. This is particularly manifested in examining the chart of the normal annual rainfall of the country. If the United States could be imagined to be of uniform elevation, it is probable that the isohyetal lines would run nearly due east and west, evenly spaced, with a slight northeasterly deflection near the Atlantic Ocean caused by increased atmospheric moisture from the source, and conversely a southwesterly trend west of the 97th meridian caused by increasing distance from the Gulf. The reader should not infer from this that heavy rainfalls owe their moisture exclusively to evaporation of ocean waters. Much of it is supplied by evaporation from land surfaces.

(2) The next important factor will be the distribution of land and water. The coastal regions proximate to the sea will surely receive greater amount of moisture than places otherwise located. Since most of the heavy storms over the interior part of the United States owes its moisture from the source of the Gulf, this effect of land and water tends to accentuate the latitude control.

(3) Difference in altitude is responsible for much variation in great storm frequency and strength, a decided decrease being noticeable in mountain regions. For instance, the Appalachian system is responsible for a pronounced southerly deflection in the isohyetal lines. The rapidly increasing elevations of the Great Plains region west of the 97th meridian contribute materially toward decreasing the normal amount of precipitation in the westward direction.

(4) The effect of mountain barriers is discernible however, only in the division of the whole United States into the parts of entirely different climates by the Rocky Mountains. It is the eastern part chiefly of continental climate that we are now concerned about.

Another lesson that we can learn from the chart of northern and southern storms arranged by seasonal occurrence, is worthwhile to be mentioned here: Over the United States there is a belt of stormy weather oscillating poleward and equatorward as the sun moves to and fro in the course of its regular migration. Northward across the country swings this storm belt as the summer sun comes north of the equator, and southward it swings in winter following the declining sun, covering the country to the Gulf of Mexico. The chart shows that the northern storms occurred during the third quarter of the years - May to July - are practically of same magnitudes and in same amount as the southern storms occurring during the first quarter - November to January. This is exactly what we may expect from the theoretical reasoning.

VII. DISCUSSIONS OF TIME-DEPTH RELATIONS.

The discussions by Miami Conservancy District of the time-area-depth relations are so instructive that they are worthwhile to be quoted. *

"From a knowledge of the causes of precipitation in interior continental regions it would be expected that the maximum 1-day period of rainfall would be preceded and followed by periods of gradually increasing and decreasing rainfall, especially during the winter months. This is generally found to be true, the maximum 1-day rainfall rarely occurring on the first or last day of the storm period."

"Another very noticeable fact is that the depth of rain which falls on the maximum day is almost always more than half the total of the storm period, regardless of the number of days in the latter. This indicates, that much higher rates of precipitation occur in periods of less than a day. The accuracy of this conclusion has been verified at individual stations".

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* The Miami Conservancy District Technical Report No.V. P. 151.

The above two facts can readily be explained if one has some acquaintance with weather maps. The "lows" travel at an average velocity of 31.7 miles per hour eastward. That is why the higher rates of precipitation always occur in periods of less than a day. It is natural that the centers of lows approach a region gradually, first coming the lower isobars (or equal pressure lines) next following the lowest pressures or centers, and lastly coming the lower isobars again. It follows, therefore, the maximum 1-day rainfall rarely occurring on the first or last day of the storm period.

"In general, there are no very great differences between the longest storm and the second or the third longest. This is more nearly true of the northern group than of the southern, and for the 1-day, 2-day, and 3-day periods than for the 4-day and 5-day periods." This point will be discussed later.

XIII. THE PROPOSED DIVISION OF VIOLENT STORM ZONES IN THE EASTERN UNITED STATES.

As explained in Art. VI, some parts of the United States are apt to have more chance of passing storms or have storms yielding heavier precipitation than other parts are apt to have. The reasons can be found from the general climatic controls described in Art. I. This leads to the introduction of an important feature climatology, the division of storm zones in the United States.

Various classifications of climates have been made for the whole world as well as for some particular parts of the world. But the present work is of a different aim. This division of zones is made from the standpoint of the frequency and magnitude of the passing violent storms. It is not a classification of climates, although the difference in the chances of passing storms between any two regions is a result of difference in climates. It is still not a classification according to the depth of precipitation, although the depth is a direct result of passing storms. Consequently, there is no single climatic classification available that can be directly utilized for the present purpose.

The classification hereby proposed is based upon an examination of the three ways of representations of the available data:

(1) The geographical distribution of rainfall types,

(2) The maximum precipitation in 24 consecutive hours in various parts of the United States. and

(3) the Geographical location of the 160 great storms studied by the Miami Conservancy District.

(1) THE GEOGRAPHICAL DISTRIBUTION OF RAINFALL TYPES.

The rainfall types here given are those of Ward's[1] based upon an examination of a large number of plotted monthly rainfall amounts for selected stations in all parts of the United States, and upon a comparison of the curves thus obtained with the rainfall types suggested by Greely,[2] Henry,[3] and Kincer[4]. Each type is illustrated by a curve showing the monthly amounts of rainfall. These curves are composites. Each one is based upon the records from several (usually five or six) stations in

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1. The climates of the United States - Ward. pp. 184-200.
2. A. W. Greely, "Rainfall Types of the United States." National Geographic Magazine Vol. 5, (1893) pp.45-58, Plate 20.

Geographical Distribution of Rainfall Types the same general district. In using a composite curve instead of the curve for a single station, the advantage is that individual local pecularities and errors arising from the topography, the altitude, the exposure of the gage, and from other local controls, are to a considerable degree neutralized. Of course it must not be expected that the curve for any individual station in any given year will agree absolutely with the type curve for the district. These composite curves show only the normal dominant type of rainfall distribution over the areas for which they have been selected as illustrations. Both the locations and monthly distributions of these types are shown in the accompanying chart.

It appears to the author that what we can learn from the rainfall types that has a bearing on our purpose are: (1) the relative annual and monthly amounts of precipitation for different types give an index to the frequencies of passing storms over the respective regions with due reference to their geographical locations.

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3. A.J. Henry. "Rainfall of the U.S., with Annual, Seasonal and Other charts", U.S. Weather Bureau Bull. D. 1897, pp. 11-13. idem, "Climatology of the U.S." U.S. Weather Bureau Bull. Q, 1906. pp.50-51.
4. J. B. Kincer, Precipitation and the Humidity Section of Atlas of American Agriculture, Part II, Climate (1922) pp. 18, 37, Fig. 13

Thus we have understood from the storm records that there are many more heavy storms recorded over the regions of the Missouri type than of the New England. Nevertheless, from the present chart, we see that the latter receives more rainfall monthly and annually than the former. This shows readily that the Missouri type has heavy storms but less in number, while the New England type has many more storms but of small magnitudes. The phrase "all roads lead to Rome" might be paraphrased into "all storm tracks lead to New England". (2) The geogrphical location of each type provides the explanations for the causes of such depth and distribution of rainfall, and thereby have some critical bearings on the expected heavy storms. Thus in the Texas coast type, there is a marked early-fall (September) maximum, which is radically different from the North Gulf Coast types. This is because the strong southeast wind from the Gulf, which is essentially monsoonal in character, interferes with the building up of summer cumuli and cumulo-nimbi clouds. In other words, connectional processes go on along the northern coast of the Gulf but are interrupted on the Southern coast. #

I.R. Tannehill, "Wind Velocity and Rain Frequency on the South Texas Coast", M.W.R. Vol.49(1921)pp.498-499.

No attempt will be made here to explain the characteristics of each type from a meteorological point of view. The readers are expected to refer to these books listed.

(2) Maximum precipitation in 24 consecutive hours. Thd data regarding heavy rainfalls during the twenty year period 1895-1914 have been charted[3].# The heaviest recorded rainfalls in twenty-four hours varied from over 10 inches along the Gulf Coast in Texas and Louisiana, where the downpours occurred in connection with West Indian hurricanes to 4 inches over the Plains and the northeastern states. (See the accompanying chart) On the north Pacific Coast the twenty-four hour amounts have not exceeded 5 inches, although the mean annual rainfall is heaviest there.

The importance of such a chart in developing the classification of storm zones is so manifest that the reader should not neglect to notice it.

The records of most intense rainfall given by the Miami Conservancy District in charts* of 1-day to 6-day durations should be more accurate than the present chart.

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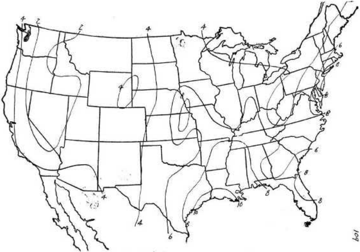
* Miami Conservancy District Technical Reports Part V. pp. 103-105, Fig. 38-43.

\# Section on Precipitation and Humidity of the Atlas of American Agriculture, Fig. 70, Text Page 42.



Maximum Precipitation in 24 Consecutive Hours.
in Inches

of 1-day to 6-day durations should be more accurate than the present chart. The period of record is from 1892 to 1916, being 5 years longer than the latter. The records show all increase in maximum depths, but the relative values and tendencies of contours do not change very much. So for out purpose, these charts are not reproduced here, although frequent reference to which has been made.

(3) The geographical location of the 160 great storms studied by the Miami Conservancy District. The 160 storms occurred during the period 1892-1918 have been plotted in the accompanying chart showing the geographical locations of the centers of action. The thirty most important storms whose time-area-depth relations have been studied by the District are marked with cross and number. The other storms show dots only.

A glance at these plottings shows readily which parts of the Eastern United States are more apt to have violent storms passing through. But these indicate the number of storms of great magnitudes above that defined. Small storms though of great number have been excluded.

Upon an examination of the above three ways of

Location of Centers of The 160 Great Storms Occurred Between [illegible] - 1916 of representations of rainfall data a division of violent storm zones in the Eastern United States is hereby proposed. (See accompanying chart). The division is rather general. Again, we should say that it is more empirical than theoretical, in considering its original basis. For instance, although meteorologists cannot explain why the "Central Mississippi Zone" as here named is apt to have more passing cyclone storms of tremendous size than the "Ohio Zone" is apt to have; still the records show that it happens to be so. Nevertheless, the establishing of the "Gulf States Zones", the "Atlantic Zones" and the "New England Zones" is based mainly upon the theoretical climatic controls over these regions as well as an observation of available records.

It should be borne in mind that the division lines of zones so drawn are not absolute. They do not represent separations of two parts of different characteristics. Except due to abrupt changes of physiographic conditions such as those between the Atlantic and Ohio Zones by the effect of Appalachian Ranges, these changes toward each side of a division line are always gradual.

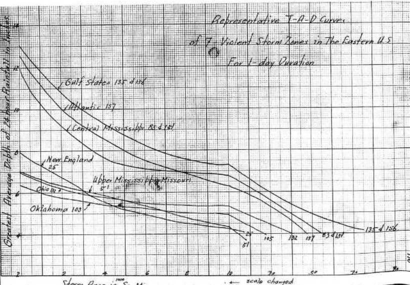
The reasons for dividing these zones in such a manner have quite fully been explained in the preceding and present articles that it might seem to the readers as a natural consequence.

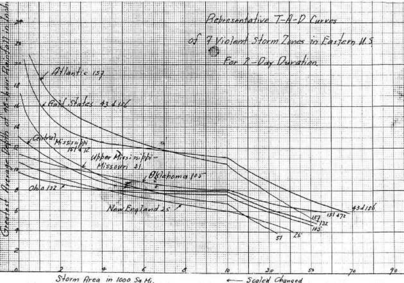
At first, the author planned to classify storms according to their tracks as described in Art. IV, but later on he felt that such a way is improper and yet not desirable. The objections are evident: (1) Storm tracks are not definite at all. Those proposed by the different meteorologists represent the general routes only and are subject to variations. (2) The tracks shift seasonally, northward from winter to summer, following the migration of the sun. Such a factor can hardly, if not entirely impossible, to be taken account of in devising a classification, and (3) The strength of storm and amount of precipitation even varies along each track. This makes such a classification of little usefulness.

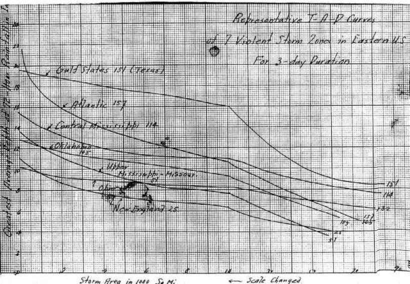
New England
Upper Mississippi-Missouri
Ohio
Central Mississippi
Atlantic
Oklahoma
Gulf States
EXPLANATORY NOTES.
Observations taken at 8 a. m., 75th meridian time.
Air pressure reduced to sea level.
ISOBARS (continuous lines) pass through points of equal air pressure.
ISOTHERMS (dotted lines) pass through points of equal temperature; drawn for every 10°.
SYMBOLS indicate state of weather: ○ clear; ◐ partly cloudy; ⊕ cloudy; R rain; S snow; M report missing.
Arrows fly with the wind.
Wind velocities of less than 10 miles an hour, and amounts of precipitation of less than 0.01 inch, are not published hereon.
Statute Miles

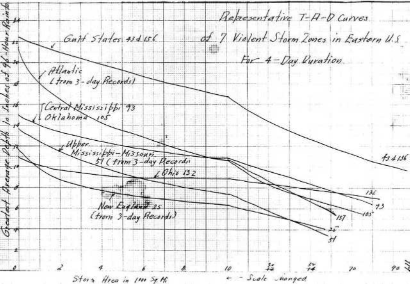
IX. STANDARD MAXIMUM TIME-AREA- DEPTH CURVES IN THE ZONES OF THE EASTERN UNITED STATES.

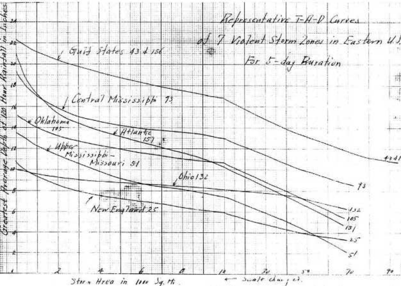
From the geographical locations of the thirty most important storms the maximum time-area-depth curves are made for each of the seven violent storm zones representing the maximum relations of the storm factors recorded during the period 1896-1916. These are plotted from the original curves made by the Miami Conservancy District on Technical Reports Part V, Fig. 94-103. The curve of each zone here drawn is, however, not necessarilty from one single storm in the zone. It consists of parts of some curves that will give maximum depths of rainfall for these respective parts. In the subsequent five charts of five lengths of duration of rainfall, the storm numbers are marked which can be referred to for their locations and times of occurrence from the charts of Arts. VI and VIII.


Representative T-A-D Curves
of 7 Violent Storm Zones in The Eastern U.S.
For 1-day Duration
Gulf States 135 & 156
Atlantic 157
(Central) Mississippi 83 & 151
New England 25
Upper Mississippi-Missouri 51
Ohio
Oklahoma 103
25
51
105
132
157
83 & 151
135 & 156
Greatest Average Depth of 24-hour Rainfall in Inches
scale changed


Rebresentative T-A-D Curves
of 7 Violent Storm Zones in Eastern U.S.
For 2-Day Duration
Atlantic 157
Gulf States 43 & 156
Central Mississippi 151 & 172
Upper Mississippi-Missouri 51
Oklahoma 105
Ohio 132
New England 25
Greatest Average Depth of 48-hour Rainfall in Inch
Storm Area in 1000 Sq Mi.
← Scaled Changed


Representative T-A-D Curves
of 7 Violent Storm Zones in Eastern U.S.
For 3-day Duration
Gulf States 151 (Texas)
Atlantic 157
Central Mississippi 114
Oklahoma 105
Upper Mississippi-Missouri 51
Ohio 130
New England 25
Greatest Average Depth of 72-Hour Rainfall in In.
Storm Area in 1000 Sq. Mi.
← Scale Changed


Representative T-A-D Curves
of 7 Violent Storm Zones in Eastern U.S.
For 4-Day Duration
Gulf States 43 & 156
Atlantic (from 3-day Records)
Central Mississippi 93
Oklahoma 105
Upper Mississippi-Missouri 51 (from 3-day Records)
Ohio 132
New England 25 (from 3-day Records)
Greatest Average Depth in Inches of 96-Hour Rainfall
Storm Area in 1000 Sq. Mi.
← - Scale changed



Representative T-A-D Curves
of 7 Violent Storm Zones in Eastern U.S.
For 5-day Duration
Greatest Average Depth of 120 Hour Rainfall in Inches
Gulf States 43 & 156
Central Mississippi 73
Oklahoma 105
Atlantic 151
Upper Mississippi-Missouri 51
Ohio 132
New England 25
43 & 156
73
132
105
151
25
51
Storm Area in 1000 Sq. Mi.
← Scale changed

120

X. USE OF THE TIME-AREA-DEPTH CURVES.

It appears to the author that the practical use of the time-area-depth curves should be as follows: First, determine (1) the storm zone to which the watershed belongs, (2) The area in thousands of square miles of the watershed, and (3) The approximate time of concentration of water flowing from the farthest end of the watershed to the point where the protection or any engineering work is to be provided. Then, by referring to the time-area-depth curves given in Art. II, the maximum depth of rainfall of a duration equal to the time of concentration in such an area of watershed within the zone can readily be picked out. This depth divided by the corresponding duration will give the average rate of intensity of rainfall.

If the watershed in question situates on or near the border of two or three zones, then the several time-area-depth curves for these zones should all be investigated. Due consideration and judgment should be made to find the suitable results which are not necessarily the extreme values among the two or three zones.

The time of concentration* is an important factor

* Of course the difficulty is that the time of concentration is in turn depending upon the intensity of rainfall.

in dealing with such a matter. What we are interested in eventually is the maximum discharge of the river channel to be provided for. This includes a time factor, or rather a rate than simply an amount. Until the time of concentration from the farthest end has elapsed the whole drainage area above the point in question will not all be taken into effect of the covering rain, consequently, the river discharge will not reach the peak. So in order to utilize the whole drainage area, the duration of rainfall should not be shorter than the time of concentration. On the other hand, it is a fact that the longer the duration, the less will be the intensity of rainfall, which will again reduce the river discharge. Therefore, the time of concentration should be taken as the duration in entering the time-area-depth curves.

This method of using the curves evidently involves a number of assumptions, and all of which tend to make the average depth over the area appear greater than would probably ever occur in a storm identical with the one applied. But in considering the case that a greater storm than any yet recorded may materially outweigh all of the above overestimates, we have to work the problem on the side of safety.

By the present method, the frequency or period of occurrences of a great storm is not determinate. The dearth of storms in each zone and the complicateness of defining the sizes make both the statistical and empirical methods impracticable. Notwithstanding this, an attempt will be made sto attack the question in a later article.

In considering the effect which a storm will produce it is necessary to take into account the topography of the area, season of the year, vegetable growth, character of surface soil, its condition and degree of saturation. These enter into the main problems of hydrology.

The time-area-depth curves here presented are not adapted for use in the design of sewer systems. In the latter, time units as small as hours and minutes, and units of area as small as acres are essential. Section II applies more particularly to rainfall over small or negligible areas.

XI. THE PROPOSED METHOD OF EXTREME FLOODING.

From considerations of the effect of great storms in producing inundation of the watersheds, another method of extreme flooding is hereby proposed. In studying such a problem, we can always analyse it into two parts. First, we determine the probable amount of precipitations brought over the watershed by the storm. Secondly, we find the river discharge from the distribution of such an amount of precipitation with respect to the watershed below. The method ~~now~~ presented is embodied in the following steps:

(1) Knowing the storm zone to which the watershed belongs, find its area, shape and orientation from the contour map of its topography. Investigate also the time of concentration.

(2) Make a list of the storms from the records that had passed over the zone in time-area-depth relations.

(3) Knowing the area of the watershed, the approximate time of concentration, figure out the depth of rainfall of each passing storm. Multiply the depth of the drainage area, gives the maximum amount of precipitation over the watershed for each storm.

(4) If only great storms have been recorded, as usually are, apply the method of arbitrarily adjusted duration curves to the amounts of precipitation determined in (3). The method is same as used in Art. VI, Part II. The duration curve is extended to find greater amounts of precipitation for rarer chances of happening. In case all storms, great or small, have been recorded for a period of years, then the statistical method can be used. Refer Part I.

(5) Arrange this amount of precipitation over the drainage area by means of isohyetals of an ideal storm with respect to the topographic contour map in such a manner as to produce maximum discharge at the point interested. This step might require some explanation. Usually a set of "time zones" representing zones of equal time of concentration should first be made. This is similar to the run-off calculations in sewerage designs. (Refer Metcalf and Eddy "Sewerage and Sewage Disposal") Then put the center of storm with highest rate of rainfall on the farthest "time zone" and the lower rates on the nearer "time zone". After several trials have been made, the discharge at the point can then be calculated. The ultimate value will evidently

be almost equal the amount of precipitation from (4) divided by duration.

The run-off coefficient is here considered equal to unity. Its determination is another big, independent problem.

It should be noticed that the present problem is quite an intricate one. If the storm center is moving from the upper to the lower parts of the watershed, the river discharge will be materially increased. Again, the relative sizes of the drainage area and storm area will have direct bearing on the result throughout the process of the method. The orientation of the axis of the watershed is another factor of prime importance. All storms with isohyetal ellipses point from due East to due North, mostly northeast, but no one points in the East West quadrangle. This is due to the effect of the prevailing westerlies. Now if the watershed axis lies in a northeast direction, it is apt to cause more floods than otherwise in the Northwest direction from a same size of storm. In one word, no statement besides general principles can here be explained. The reader should try to visualize this himself.