THE ANALYSIS OF
THE RAINFALL-RUNOFF CORRELATION

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THESIS

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Synopsis

This paper points out the necessity of securing hydrographs rather than merely maximum and minimum stream flows for engineering uses. As most stream-flow observations available extend over a comparatively few years, there is an urgent need for an accurate method of computing hydrographs from rainfall and other physical data.

The proposed theories and methods herewith presented are based upon the fundamentals of the scientific methods of analysis and synthesis. Time and Space, the basic variables of any natural phenomenon in motion have been fully considered. The underlying principle is to detect all physical characteristics pertaining to a drainage area that are impossible to be obtained by measurements, from the hydrograph itself. Any irregular feature of an observed hydrograph is to be explained with reasons. The usual method of computing the several kinds of water losses such as evaporation, transpirations, etc., to obtain the net runoff has been entirely abandoned in the present research. Part I treats of the introductory comments of these said principles about the theories and methods followed.

In Part II, the proposed theory of instantaneous hydrograph is presented from which time contours can be determined on the map. With the time contours known, all drainage area characteristics are completely revealed. Besides, hydrographs can be reproduced even from non-uniform rainfalls. There are several other useful methods

presented in this part. The ordinarily used unit-graph method is also criticized. However, all methods given in this part are based upon three assumptions: (1) There is no water lost throughout its trip to the measuring point so that all rainwater is drained from the surface of the basin; (2) The ground conditions are invariable with the rising of river stages so that the instantaneous hydrograph is constant; and (3) A uniform rainfall covers the entire drainage area when used for analysis.

Part III: "The Rate of Water Losses," takes into account the assumption (1) in Part II. A method of determining the amount of water losses by means of the "differential hydrograph" is proposed.

Part IV: "An Exact Analysis," takes into account the remaining two assumptions in Part II. Thereby, the theoretical part of the methods have been completed.

In Part V: "Problems Involving Channel Storages," a method of eliminating the effect of channel storage is proposed, and criticisms are given to the Horton's method. A method of finding the ground water depletion curve and another method of separating recession curves are also presented. These methods are useful in the actual solution of problems.

In Part VI, the details of solution are presented with an illustrative example. It describes the procedure to be followed in the analysis and synthesis of hydrographs. The West Branch of Salt Fork Basin at Urbana, Illinois has been chosen for illustration. The results have met the tests of the theories and methods established by the writer.

CONTENTS

I - INTRODUCTION

1 - Complexities of the Hydrologic Phenomena

Hydrologic Phenomena are very complicated. A drop of rainfall, after it is precipitated, and before it reaches well-defined stream channels, is subjected to the action of numerous agencies, including evaporation, transpiration, infiltration and surface runoff through systems of trickles and streamlets. These agencies vary widely under different conditions. A brief description of the factors affecting them will be given.

Consider a rainfall superimposed upon a given drainage basin. The meteorological conditions are not so simple as we ordinarily assume. The intensity of rainfall not only varies with different points in the basin, but also changes as time goes on. Thus a set of isohyetal lines may hold only for the particular instant it refers to, and there may be a different set for another instant.

In falling down near the ground surface, a portion of the precipitation is caught and held by foliage, twigs and branches of trees, shrubs and other vegetation; and is afterwards evaporated from these surfaces. This is grouped under the head "interception".

After it falls on the ground, the numerous agencies come into action. The rôle of infiltration is rather intricate. Beginning at the instant the rain touches the ground, the rate of infiltration, in general, decreases from a maximum gradually to a more or less constant value as a result of the combined action of capillary and gravitational flow. Moreover, this rate is by no means uniform at all points throughout the basin. It varies according to the soil constituents and other conditions at different locations. Therefore, the variables for the infiltration rate, just the same as those for the intensity of rainfall, are time and locality. And as we determine a locality by two coordinates, it is seen that the problem is a three-variable one.

The part of water entering the ground soil travels down to the water table and accrues to the ground water flow. The rate of this flow varies with the sizes of the pores between the soil grains, the head impelling the flow, and the soil temperature. It is so slow that it takes considerable time before the water seeps out to the stream channel. On the other hand, that part of the ground water held above the water table as capillary water is constantly absorbed by the roots of trees and vegetation and is eventually vaporized from the breathing pores, or stomata, of the leaves and other vegetable surfaces. The dominant factors affecting the rate of transpiration are temperature, humidity of the atmosphere, soil moisture and the character of vegetation.

That portion of the precipitation which is not absorbed by the surface soil or lost by evaporation and interception flows to stream channels over the surface of the ground. Before this runoff begins, however, there is a surface layer of water which accumulates on the ground and which may be described as initial detention. It is made up of two principal elements: (1) water required to fill depressions; (2) water which accumulates in transit during the filling of the depressions overland to the stream channel. The water contained in the depressions themselves, up to their overflow levels does not in any way contribute to the volume of surface runoff. An equivalent amount remains in the depressions at the end of surface runoff and is afterwards disposed of by infiltration to the soil or by evaporation.

When the initial detention has been filled, the rainfall in excess of infiltration flows toward the outlet or stream. Since the volume of supply increases from the water-shed divide to the stream bank, the depth of surface detention increases in the same direction. The rate of outflow at any given time is a definite function of the maximum depth of surface detention along the margin of the outlet channel. This results from the fact that direct surface runoff is essentially the same as the flow in a channel, and the rate of flow is an approximate function of depth.

After the water has entered the channel, the problems of channel detention and channel flow are involved. The stream channel functions as a reservoir which temporarily checks the flowing water as the depth of the stream is raised. This is seen from the fact that the flood hydrograph at a given station in a channel is modified as it passes down to another station. The peak of the hydrograph is lowered, while the base is spread wider. The effect is to reduce the rate of discharge by allowing a longer time of flow.

Throughout the hydrologic cycle which we have discussed, the work of evaporation takes place. It is the process by which surface or sub-surface water is converted to atmospheric water. This takes place from water surface as well as from ground surface.

During short intense rains, evaporation may generally be considered negligible since the relative humidity attains a very high percentage. Other factors influencing the rate of evaporation besides relative humidity, are temperature, atmospheric pressure and wind velocity.

2. Preliminary Considerations on the Rainfall-Runoff Correlation

It is thus seen that our consideration of the hydrologic phenomena has led into a maze of complexity. If we were to extend the analysis far enough, it would be found to reach throughout the whole field of natural phenomena, a digestion of which requires the knowledge of many branches of science. Nevertheless, one remarkable fact is that it has a system. Each process has to move with precision toward a set destination, and therefore it must be working in accordance with principles that observation and study will reveal.

The earlier splendid works of A. F. Meyer, D. W. Mead and others were to measure the various agencies individually, as precipitation, evaporation, transpiration, percolation, etc., and to combine them to obtain the average monthly discharge. For an accurate determination of the latter, the method requires much hydrologic skill and judgement that the average engineer can hardly possess. ~~Even it is possible,~~ Besides, we are not satisfied at all with the long-term monthly discharges. It is the continuous hydrograph that we need ultimately. Also, statistical methods applied to the recorded river discharges have been used to estimate the maximum and minimum values corresponding to certain frequencies of occurrence.

Before describing the methods of obtaining the continuous hydrograph, there are some preliminary considerations that should be discussed here.

(1) Although river discharge is the result of the works of many agencies none of which can be estimated to a desirable accuracy, we as engineers, are not directly interested in the quantities of these individual agencies, as evaporation, transpiration, etc., except indirectly for obtaining the river discharge. It suffices to group them together under the general head "rate of water losses," which, during the raining period, equals the intensity of rainfall minus the rate of runoff.

(2) It is evident that with the same rainfall superimposed over a drainage area, the resulting hydrographs will be the same, providing that the surface cover and other related conditions remain unchanged. It follows, therefore, that if we can correlate the rainfall and runoff successfully, the problem of hydrology will be reduced to only one of meteorology and climatology.

(3) The statistical method is fundamentally a device to find the probability of happening of an event which has no definite order of occurrence, or at least none that we know of. One event, however, may be composed of several constituent events, and the probabilities of the event in percent is the product of the probabilities of the constituent events in percent. Some of these events may sometimes be of definite order of occurrence, or the

chance of happening is 100 percent. In such case, these events may be eliminated and disregarded, so that we can observe more accurately the corresponding values of frequency against magnitude of the remaining events. Such is the case in the problem of determining the stream discharges. After the phenomena of rainfall and runoff have been well correlated, statistical method can be applied directly to the phenomena of precipitation, i.e., intensity, area and location of the area. Provision has to be made, of course, that other conditions remain the same in the same seasons of different years.

(4) It should be noted that in any part of the world the rainfall records have been kept much longer than the records of stream gaging. This arises from the facts that the need of stream gaging has been felt only in the late decades and that rain gages are much cheaper to install and maintain. Statistical methods can be expected to give reasonable results only when the recorded data are abundant enough, otherwise, it will yield apparently ambiguous ones. The dearth of stream gaging data has long been experienced by many engineers as a serious handicap in applying statistical method to river discharges. J. J. Slade, Jr., after himself having devised a type of statistical method, mentioned in his conclusion of "The Reliability of Statistical Methods"[1] that "In the writer's opinion the statistical method, in whatever form employed, is an entirely inadequate tool in the determination of

1. Geological Survey, Water Supply Paper 771, p. 432.

flood frequencies." Moreover, there are rivers with no stream gaging records at all, in which case, the rainfall records will be the only guide.

From the above considerations, it is evident that the correlation of the rainfall and runoff phenomena would be a great assistance to the present status of engineering hydrology, and even might be the only logical method of solution. This is the purpose of the present research. It is expected that all characteristics of a drainage area, such as the slope and configuration of the basin in affecting the velocity of flow, the conditions of ground in controlling the rate of water losses, etc., are to be differentiated and reflected from the known hydrographs corresponding to the given rainfalls.

3. Two Scientific Methods of Solution

The whole problem of rainfall and runoff correlation is actually one of transferring the rates of flow through a cross section at the outlet which is vertical, to the rates of rainfall, or intensities over a plane of watershed which is horizontal; or vice versa. It can, therefore, be separated into two processes: the Analysis of Hydrograph and the Synthesis of Hydrograph. The former deals with the differentiation of a hydrograph into the various constituent factors that compose it. The latter, with these factors known from the analysis to combine them for a given rainfall to obtain the required hydrograph. The required data, in the process of analysis, will be a contour map of the drainage basin, some corresponding rainfalls and hydrographs at the outlet. In the process of synthesis, of course, the rainfall over the same basin has to be given.

As seen from the foregoing descriptions of hydrologic phenomena, both rainfall intensity and the rate of water losses vary with time and space which are the two fundamental independent variables of any natural phenomenon in motion. However, we can reduce space to area, inasmuch as the known configuration of a watershed serves as a given condition. Moreover, it will be shown later that water always moves in definite routes or flow lines such that the variable area is only a matter of distance. Consequently, time and distance are the only independent variables.

In the analysis of a hydrograph, we trace from the hydrograph as a culmination back to the rainfall as its origin, through the operations of the numerous hydrologic agencies. As just stated, the phenomena are a function of the time elapsed and the distance up from the outlet; or $q = F(t,x)$, in which q denotes the rate of rainfall excess, t the time, x the distance and F the function. Mathematically, when we plot such an equation of surface into the Cartesian coordinates, we either, by assigning a value to x, calculate the values of F for several values of t in the particular x - plane, and do the same for other sets of F by assigning other values of X; or else calculate sets of F where x varies on different t - planes. This very conception of the two ways of plotting an equation of surface leads to the two methods of rainfall and runoff correlation. In the former, by tracing distance with different x-planes, we can find the hydrograph for any distance from the outlet. Thus, when $x = x_1$, (x_1, a definite value) we obtain the hydrograph expressed in discharge against time, $Q = F(t_1, x_1)$. This is really the essence of Horton's method of solution, although in his book[2] he did not have this mathematical conception. In his analysis, Horton first finds the hydrograph with the effect of channel storage eliminated, which actually represents the hydrograph of water flowing through a long sigzag section that hugs along the water edges of the channel. (See discussions in Sections 21 and 22). Next he eliminates the effects of surface detention to obtain the rate of rainfall excess. To the latter, he adds the rate of infiltration previously found and finally arrives at the rainfall intensity.

The second method of plotting the function $Q = F(t, x)$ by assigning consecutive values of t instead of x, suggests to the writer a different method of solution. Consider a superimposing rainfall with known isohyetals at t = o last an instant dt, and find the resulting hydrograph due to this instantaneous rainfall. In the next instant dt, given another set of isohyetals, find another hydrograph due to the rainfall of this instant. By extending this argument from t = 0 to the duration of rainfall and setting the hydrographs due to each instantaneous rainfall in their proper positions of the t-axis, the summation of the ordinates of these hydrographs at the same abscissa will give the required hydrograph due to the given rainfall. This is the synthesis of hydrograph. Conversely, the analysis of hydrograph is to find the hydrograph due

2. R. E. Horton: Surface Runoff Phenomena, Part I.

to a given instantaneous rainfall. These are the essence of the writer's method.

It can be shown that the first method, to tracing by distance, is rather difficult and cumbersome. As rain falls on all parts of an area, instead of coming from one source of headwater, the stream discharge varies from place to place along the channel. Yet Horton's method does not provide for the variations of rainfall and infiltration rates with respect to either time or locality. It can be applied only to very small areas. ~~(a few acres)~~ Therefore, it is not a good method of solution.

4. Definitions of Terms

Rainfall excess - That part of the rain of a given storm which falls at intensities exceeding the rate of infiltration.

Initial detention - The volume of water on the ground, either in depressions or in transit, at the time active runoff begins.

Initial moisture deficiency - The amount by which the actual water content of a given soil at the beginning of rainfall is less than the total amount of soil moisture required for the beginning of rainfall excess.

Rate of water losses - The total amount of water lost in a unit time due to all possible agencies, such as infiltration, evaporation, transpiration, etc.

Channel storage - The volume of water in definite stream channels above a given measuring point at a given time during the progress of runoff.

Recession curve - That part of a hydrograph where the discharge decreases with increase of time.

Ground water depletion curve - A curve expressing the relation between time elapsed and rate of ground water supply to streams in a given drainage basin during periods when no direct surface runoff is taking place.

"Platform" - A platform-like feature of the part of a hydrograph where discharges are practically invariable with respect to time.

"Hump" - A hump-like feature of the part of a hydrograph where the curve first slopes down gradually and then turn down suddenly as time goes on.

II - Drainage Area Characteristics

5. The Theory of Instantaneous Hydrograph

An instantaneous hydrograph is one due to an instantaneous rainfall falling over the drainage area above the section of river where the discharge measurements are made. Interpreted mathematically, an instantaneous rainfall means that its duration approaches zero as a limit.

Consider a uniform rainfall of unit intensity, say, one inch per day, covering the entire drainage area and falling in an infinitesimal time, dt. Assume that there is no water lost throughout its trip to the outlet so that all rainwater is drained from the surface of the watershed. Consequently, the rate of rainfall excess is equal to the intensity of rainfall. Following such an instantaneous rainfall, the hydrograph representing the flow in the main stream channel shows the run-off increasing to a maximum value and then subsiding to the value it had before the storm. In Fig. 1, O M N represents such a graph. O N = too is the instantaneous flood period or the time of concentration of flow from the remotest point of the basin to the measuring point. tpo is the time that elapses until the peak flow is reached.

The theory of instantaneous hydrograph[3] provides that the shape of the graph when developed is entirely definite for the particular watershed concerned, independent of the intensity, duration and frequency of any storm that might occur over the

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3. This theory is not exact. It is presented simply for convenience of explanations. See Part IV, An Exact Analysis.

area, with the conditions that the rainfall is uniform and that the rate of water losses is nil. A different intensity of rainfall changes the ordinates of the graph proportionally, but not the time of concentration, too, nor the peak flow period, tpo.

The theory is based upon the fundamental axiom that any physical phenomenon repeats itself if under exactly the same conditions. Therefore, at the times when the ground conditions are constant, a uniform rainfall of a given intensity can yield one and only one type of instantaneous hydrograph. For different intensity, the proof of the theory will be given in Section IV.

With the instantaneous hydrograph given for a particular area, the hydrograph from any known storm can be computed. The procedure is simply a process of summation of the sequence of occurrences. By superimposing the graphs one by one, each moved to the right from the previous one by a distance dt until the end of the rainfall period, a hydrograph is obtained for the runoff due to the storm of given duration and intensity. Fig. 1 shows the constituent instantaneous hydrographs in their respective positions. The ordinate of any point in the hydrograph is equal to the sum of the ordinates of the instantaneous hydrographs at the same time. Evidently, dt as shown is supposed to be infinitesimal, while the number of the instantaneous hydrographs should be infinite.

Let D be the duration of any uniform rainfall; to, the total flood period and tp, the time of peak flow of the hydrograph.

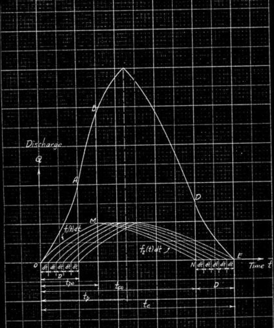

Fig. 1.
Summation of Instantaneous Hydrographs.

It is seen from Fig. 1 that

$$t_o = t_{co} + D \qquad (1)$$

The whole hydrograph O A B C D E is not a curve of continuous equation but it can be divided into five parts, each of which is continuous. The formulas for each of the five curves are derived in the following.

Let t be the time elapsed since the beginning of rainfall; Q, the discharge at the outlet at the time t; and $f_1(t)$ and $f_2(t)$ represent the ordinates of the instantaneous hydrograph with two parts discontinuous at the peak M.

For the curve O A, where $D > t > o$,

$$Q = \int_o^t f_1(t)\, dt \qquad (2)$$

For the curve A B, where $t_{po} > t > D$,

$$Q = \int_{t-D}^t f_1(t)\, dt \qquad (3)$$

For the curve B C, where $t_{po}+D > t > t_{po}$,

$$Q = \int_{t-D}^{t_{po}} f_1(t)\, dt + \int_{t_{po}}^t f_2(t)\, dt \qquad (4)$$

For the curve C D, where $t_{co} > t > t_{po}+D$,

$$Q = \int_{t-D}^t f_2(t)\, dt \qquad (5)$$

For the curve D E, where $t_o > t > t_{co}$

$$Q = \int_{t-D}^{t_{co}} f_2(t)\, dt \qquad (6)$$

In fact, the instantaneous hydrograph can be taken as a single continuous curve, f(t). Then the hydrograph consists of only three parts, with the middle part A B C D represented by the equation,

$$Q = \int_{t-D}^t f(t)\, dt \qquad (7)$$

The limits of integrations used in the above equations will become obvious if the readers bear in mind that the number of instantaneous hydrographs cut by an ordinate erected on the curve A B C D has to be equal to the duration of rainfall in units of dt.

For continuous rain lasting longer than the time of concentration of the basin, the peak flow after that time will, of course, no longer increase, but will maintain at the same runoff rate until the end of the rainfall. This, however, seldom happens except for very small drainage areas.

6. Determining the Instantaneous Hydrograph

With the instantaneous hydrograph given, a hydrograph from rainfall of known intensity and duration is constructed by a process of summation or integration. Conversely, an instantaneous hydrograph can be constructed from a given hydrograph by a process of differentiation. The following equations are established by differentiating equations (2) to (6) with respect to time.

For the curve O A, where $D > t_0 > 0$,

$$f_1(t) = \frac{dQ}{dt} \qquad (8)$$

since $\frac{dQ}{dt} = 0$ when $t = 0$.

For the curve A B, where $t_{po} > t > D$,

$$Q = \int_{t-D}^{t} f_1(t)\, dt = \int_0^t f_1(t)\, dt - \int_0^{t-D} f_1(t)\, dt.$$

Differentiating, we have,

$$\frac{dQ}{dt} = f_1(t) - f_1(t-D)$$

Therefore, $f_1(t) = \frac{dQ}{dt} + f_1(t-D)$, (9)

For the curve D E, where $to > t > too$,

$$f_2(t-D) = \frac{dQ}{dt} \tag{10}$$

since $f_2(t) = o$ when $t = too$.

For the curve C D, where $too > t > tpo+D$,

$$f_2(t) = \frac{dQ}{dt} + f_2(t-D) \tag{11}$$

For the curve B C, where $tpo+D > t > tpo$, Equation (4)

$$Q = \int_{t-D}^{tpo} f_1(t)\, dt + \int_{tpo}^{t} f_2(t)\, dt.$$

can serve as a check on the values of $f_1(t)$ and $f_2(t)$ determined from the previous equations.

The above process of differentiation can be done either graphically or analytically.

For rainfall of continuously varying intensity, it is much more complicated to determine an instantaneous hydrograph. This will be treated in Part IV, An Exact Analysis.

The extent and accuracy of the available data usually determines the degree of elaboration of the process to be followed and the percentage of errors in the results to be expected. If there is an automatic rain gage within or nearby the drainage basin and a water level recorder at the gaging station, then simultaneous continuous curves of both the rainfall and the runoff can be obtained. The method of instantaneous hydrograph will be of maximum value.

The ordinary data available in this country are however, limited to the published records of the United States Weather Bureau and the United States Geological Survey. The Water Supply Papers of the Geological Survey give only the daily average records of river discharge. The Weather Bureau publishes the records of the total rainfall depth falling in a day. There is no indication at all of the actual duration of rainfall, or whether the rain is continuous. Without this information, it would seem at first instant that the applications of the equations derived above would be impossible.

However, a method has been devised for finding the instantaneous hydrograph even though the duration of rainfall is unknown. By this means, the ordinary available data can be utilized. The procedure is to compute $f_1(t)$ and $f_2(t)$ by assuming several values of rainfall durations and to choose the one which best fulfills both of the two conditions: (1) Values of $f_1(t)$ computed by extending t beyond tpo should correspond or at least approximate to those of computed $f_2(t)$ and vice versa. (2) The expression, $\int_0^{t_{po}} f_1(t)\,dt + \int_{t_{po}}^{t_{oo}} f_2(t)\,dt$ representing the area included between the instantaneous hydrograph and the time axis should be equal to the intensity of rainfall or to the rate of rainfall excess when water losses are considered. The example in Section 7 will illustrate the method.

In finding the instantaneous hydrograph of a drainage area, the set of rainfall records and the corresponding hydrograph used should be carefully selected. The rainfall shall possess the following characteristics: (1) The intensity is very high in order to reduce the percentage of error in itself as well as indirectly in discharge measurements. A one-day record is preferred so that it is more probable that the rainfall is continuous. (2) The storm selected is so large as to cover several times the given watershed. Then it is probable that the rainfall is uniform.

7. An Illustrative Example
- The Big Muddy River at Plumfield, Ill.

The Big Muddy River at Plumfield, Illinois has a drainage area of 753 square miles. Daily rainfall records of the United States Weather Bureau are available at Mount Vernon, in the center of the upper portion, and at Benton, near the center of the lower portion of the drainage area. The records of the Mount Vernon station apply to 43 per cent of the area, and the Benton records to 57 per cent. The average daily flows of The Big Muddy at Plumfield are contained in the Water Supply Papers of the United States Geological Survey.

A search of records shows a rainfall on March 6, 1926 which fairly possesses the characteristics as described in Section 6. The rainfall records and discharge readings are listed in Table I and platted in Fig. 2.

In Fig. 2 the depletion curve is determined by extending the hydrograph from the initial flow on the day before the storm arrived. These values are taken down and entered into Column III of Table 1. In Column IV, the net runoff is the difference between the observed runoff and depletion flow. The sum, 6300 c.f.s./

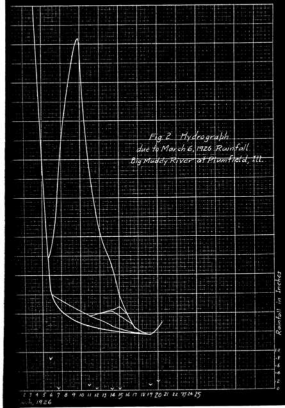
Fig 2 Hydrograph
due to March 6, 1926 Rainfall.
Big Muddy River at Plumfield, Ill.
Rainfall in Inches
2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
…rch, 1926

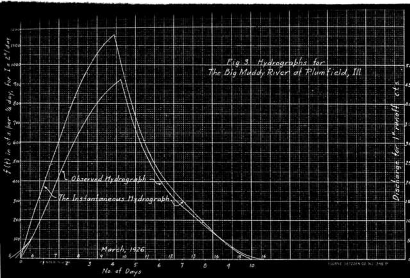
Fig. 3. Hydrographs for
The Big Muddy River at Plumfield, Ill.
f(t) in c.f.s. per ½ day, for I = 2"/day
Discharge for 1" runoff c.f.s.
Observed Hydrograph
The Instantaneous Hydrograph
March, 1926
No. of Days

day or 234 in. mi. represents the total runoff due to the 0.82 in. rainfall. It is equivalent to 0.311 in. rainfall excess. Dividing the values of Column IV by 0.311, we obtain the discharges for 1 in. runoff of Column V. The latter will be used to find the instantaneous hydrograph due to 1 in. runoff. Column IV can also be used to find the instantaneous hydrograph due to 0.311 in. runoff and then divide the results by 0.311 to obtain that due to 1 in. runoff. This follows more closely to the theory as established, but the process now employed will give the same results as the durations of rainfall are same. This is because $\int 0.311 \times f(t)\, dt = 0.311 \int f(t)\, dt$. But it should not be mistaken that the discharges for 1 in. runoff from rainfall of a different duration can also be found by dividing the values of Column IV by 0.311. Then it will not accord with the theory of instantaneous hydrograph.

The values of Column V are then platted on cross-section paper. (Fig. 3) A smooth curve is carefully drawn through them. The point on March 14th is apparently erroneous as the curve does not pass through it. This probably arises from errors in estimating deductions of flows due to minor rainfalls. These values of discharges are again taken down from the curve at intervals of one-quarter day, and entered into Column III, Table 2.

As the duration of rainfall is unknown, the method outlined in Section 6 is followed in finding the right instantaneous hydrograph. Three durations are assumed, 24, 18 and 12 hours. This makes the zero at the outset of the observed hydrograph located at different points on the time axis. (See Fig. 3) Columns III, IV

Table 1 - Calculations of Surface Flows

(Runoff records from U.S. Water Supply Paper No. 625)

Date (1926)	Observed Runoff c.f.s.	Depletion Flow Deductions c.f.s.	Net Runoff c.f.s.	Discharge for 1" run-off c.f.s.	Day	Rainfall, in. Mt. Vernon 43%	Rainfall, in. Benton 57%	Rainfall, in. Wt. Av. 100%
Feb.28	2650	2650						
Mar. 1	3060	3060						
2	2960	2960						
3	2570	2570						
4	1970	1970						
5	1220	1220		0	0			
6	706	536	170	550	1	0.82	0.82	0.82
7	1050	480	560	1800	2	.08	T	.04
8	1460	460	1000	3220	3	T		T
9	1720	430	1290	4150	4			
10	1740	410	1330	4270	5		T	T
11	1200	390	810	2600	6	.12	.14	.13
12	923	400	520	1670	7	.08		.04
13	754	414	340	1090	8	T		T
14	658	418	240	770	9	.05		.02
15	493	440	50	160	10	.06		.03
Mar. 16	390	390	0	0	11			
17	325	325	6300	20280 c.f.s./ day				
18	301	310	÷ 26.9 = 234 in. mi.			.26		.12
19	289	289						
20	325	325				.17	.25	.21

Runoff = $\frac{234}{753}$ = 0.311"

Rainfall = 0.82"

Water Losses = 0.51"

Table 2a - Computations for Instantaneous Hydrograph, $f_1(t)$

Date 1926 March	No. of Day	Q, c.f.s. for 1" runoff for D =			$\frac{dQ}{dt}$, c.f.s./ ½day for D =			$f_1(t)$, c.f.s./½ day for D =		
		24 hr.	18 hr.	12hr.	24 hr.	18 hr.	12 hr.	24 hr.	18 hr.	12 hr.
I	II	III	IV	V	VI	VII	VIII	IX	X	XI
5	0	0	0	0	0			0		
					35			35		
¼		35	0	0		0			0	
					95	80		95	80	
½		130	80	0			0			0
					170	210	240	170	210	240
¾		300	290	240						
					250	260	310	250	260	310
6	1	550	550	550						
					270	Same as the left Column	Same as the left Column	305	350	510
¼		820	Same as the left Column	Same as the left Column						
					310			405	520	620
½		1130								
					320			490	580	830
¾		1450								
					350			600	700	970
7	2	1800								
					370			675	890	1200
¼		2170								
					380			785	960	1350
½		2550								
					360			850	1060	1550
¾		2910								
					310			910	1200	1660
8	3	3220								
					280			955	1240	1640
¼		3500								
					250			1035	1310	1910
½		3750								
					210			1060 8620	1410	2050
¾		3960								
					190			1100	1430 12200	2100
9	4	4150								
					160			1115	1470	2210
¼		4310								
					140			1175	1550	2240
½		4450								
					120			1180	1550	2330 23950
¾		4570								
					-290			810	1180	1950
10	5									
		45755	45660	45530						

Table 2b - Computations for Instantaneous Hydrograph $f_2(t)$

Date 1926 March	No. of Day	Q c.f.s. for 1" runoff for any D			$\frac{dQ}{dt}$ c.f.s./¼ day for any D			$f_2(t)$ c.f.s./¼ day for D =		
								24 hr.	18 hr.	12 hr.
I	II	(III	IV	V)	(VI	VII	VIII)	IX	X	XI
16	11		0							
						20				
¼			20							
						40				
½			60							
						50				0 20
¾			110							
						80			0 20	40
15	10		190							
						80		0 20	40	70
¼			270							
						100		40	50	120
½			370							
						110		50	100	150
¾			480							
						110		80	120	220
14	9		590							
						120		100	150	260
¼			710							
						110		140	210	330
½			820							
						130		160	230	380
¾			950							
						140		190	270	440
13	8		1090							
						120		220	320	510
¼			1210							
						150		250	360	580
½			1360							
						150		290	410	630
¾			1510							
						160		330	440	730
12	7		1670							
						180		340	510	780
¼			1850							
						190		400	560	890
½			2040							
						260		440	600	960
¾			2300							
						300		490	690	1080
11	6		2600							
						330		520	750	1220
¼			2930							
						400		590	860	1380

Table 2b (Continued)

t		3330	450	700	990	1550
t		3780	500	790	1080	1780
10	5	4280	290	850	1260	2000
						16120
t		(4570)		990	1440	2300
t				1150	1580	2290
t				1290	1550	
					14590	
9	4			1140		
				11560		
		34520				

	III	IV	V
Table IIb	34520	34520	34520
" IIa	45755	45660	45530
Runoff, c.f.s.-4 day	80275	80180	80050
" c.f.s. day	20069	20045	20013

	III	IV	V
Table IIb	11560	14590	16120
" IIa	8620	12200	23930
Rate of Runoff, c.f.s.	20180	26790	40050
	x 24	x 12	x 12
	24	24	24
Runoff, c.f.s. day	20180	20093	20025
Actual Runoff c.f.s. day	20069	20045	20013
Discrepancy, day	111	48	12

and V of Table 2a shows different values of discharges at the outset for the three durations. $\frac{dQ}{dt}$ in units of c.f.s. per quarter day given in Columns VI, VII, and VIII are the differences between the consecutive intervals of Columns III, IV and V respectively. The values of f(t), Columns IX X and XI are then calculated according to Equations (8) to (11). For instance, in Column IX, when t is within the 2d hour duration, $f_1(t) = \frac{dQ}{dt}$ as given by Equation (8). After that, Equation (9) gives $f_1(t) = \frac{dQ}{dt} + f_1(t-D)$. When t is from 1 and 1¼ day, $\frac{dQ}{dt} = 270$, while $f_1(t-D) = 35$, giving $f_1(t) = 305$. Similarly, when t is from 1¼ to 1½ day, $\frac{dQ}{dt} = 310$, while $f_1(t-D) = 95$, therefore, $f_1(t) = 405$.

Table 2b shows computations of $f_2(t)$ from the other end of the hydrograph. Here Equations (10) and (11) are used. The zero points of $f_2(t)$ are located at the times as provided by Equation (1). In both Tables 2a and 2b, calculations for some points are carried beyond the peak flow.

To determine the right duration of the rainfall and the correct instantaneous hydrograph, the two principles mentioned before are applied. The sums of the observed flows in each of the Columns III, IV, and V of the both tables divided by four represent the total amounts of rainfall for each of the three durations in units of c.f.s. x day. (See bottom of Table 2b) On the other hand, the sums of Columns IX, X, and XI, representing the areas of the instantaneous hydrographs, are the rates of runoff in c.f.s. These, multiplied by their respective durations, should be also equal to the total amounts of runoff, (in c.f.s. x day) which should check the respective sums of the observed flows. It is found that for D = 24 hours there is a difference of 111 c.f.s. x day; for D = 18 hours, 48 c.f.s. x day; and D = 12 hours, 12 c.f.s. x day. At the same time, values of $f_1(t)$ computed for t beyond tpo are nearer to those of $f_2(t)$ for the 12 hour duration than the other two. Consequently, f(t) computed by assuming D = 12 hours is adopted as the instantaneous hydrograph for the watershed, although the actual duration of this rainfall is probably 11 hours. Its intensity of rainfall is one inch in 12 hours, or two inches a day. For unit intensity of one inch per day the values of f(t) in Table 2 should be divided by two.

8. An Illustrative Example for Computing Hydrograph from an Instantaneous Hydrograph Due to Uniform Rainfall

The instantaneous hydrograph method can be used to determine a flood hydrograph corresponding to certain assumed frequency. There, the amount of water lost is usually neglected. The problem divides itself into two classes: (1) The intensity of rainfall is uniform and constant over the entire watershed, and (2) it is not uniform with respect to either the locations within the area or the time during the storm period. The present section is devoted only to the former class. The latter will be treated in Section 11.

For constant and uniform intensity of rainfall, discharges are computed from Equations (2) to (6). A storm of known duration, intensity, area and frequency is given. The general process of solution is as follows: (1) Determine the frequency of the storm or flood to be provided for; (2) From meteorological studies on the "time-area-depth" relations of storms passing that general region, select several sets of corresponding durations (or times) and intensities (or depths) for the given drainage area above the outlet; (3) With these sets of data, compute the several flood hydrographs by means of the given instantaneous hydrograph and select the greatest one.

An example is shown in Table 3 and Fig. 4 for the Big Muddy River at Plumfield, Illinois. The rainfall data are taken from the writer's "Representative Time-Area-Depth Curve for the Central Mississippi Violent Storm Zone."[4] The one selected is Storm 114 of the data collected by the Miami Conservancy District.[5] It occurred on October 4-6, 1910 and covered the lower Ohio River and a portion of the Mississippi Valley, extending from the west Arkansas to northern Ohio. For a three day duration, over a 753 square mile drainage area, the rainfall was about 12 inches, or an intensity of 4 inches per day. The frequency of such a storm, although unknown, would probably be much longer than 100 years. The runoff coefficient is taken as 100%.

In Table 3, the values of f(t), Column II, are taken from Fig. 3. Columns III and IV are self-evident. Column V is obtained from the difference between the Columns III and IV, as

4. W. L. Huang: "A Study on Excessive Precipitations," p. 118, Thesis, Cornell University, Ithaca, N.Y.
5. "Storm Rainfall of Eastern United States," Technical Reports Part V, Miami Conservancy District, Dayton, Ohio, p. 163.

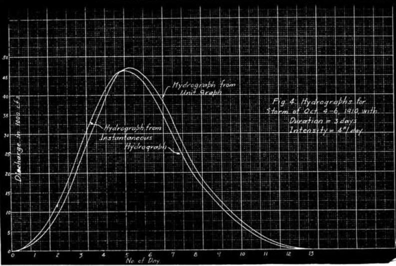

Fig 4. Hydrographs for Storm of Oct. 4-6, 1910, with Duration = 3 days Intensity = 4"/day

shown by Equation (3). Column VI doubles Column V. Discharges given in Column VII are found by multiplying by 4 the Column V of Table 1. These are plotted in Fig. 4.

9. The Significance of an Instantaneous Hydrograph

Besides what has been described, the theory of instantaneous hydrograph has a more important significance to prove its great usefulness to the science of hydrology. It is this significance that makes the later development of the present research possible.

Let us consider again a rainfall of uniform intensity $\frac{H}{D}$ with respect to both time and location, covering entirely a given drainage area. The waters at all points of the area flows through trickles, streamlets, tributaries and then the main channel toward a common destination, i.e., the outlet or measuring station. Some waters from different points, though taking different paths, will finally meet at the outlet at the same time. If a line is drawn through these points from which waters take the same time to travel to the outlet, such a line is called a time contour. Fig. 5 shows some time contours drawn on the drainage area.

Let the velocity of flow of water at any point be V or $\frac{ds}{dt}$, and suppose that the time contours for the area are already known, the length of any one of which is denoted by l_t corresponding to time t. Within the duration of rainfall D, water advances a distance $\int_0^D V\,dt$, or V D for constant velocity. Now consider any two time contours t and (t-D). Any drop of water which falls on a point on the latter contour at the beginning of rainfall will

Table 3 - Computing Hydrograph from an Instantaneous Hydrograph

Day, t	f(t) c.f.s./¼ day I=2"/day	$\int_0^t f(t)\,dt$ c.f.s. I=2"/day	$\int_0^{t-D} f(t)\,dt$ c.f.s. I=2"/day	Q c.f.s. I=2"/day	Q c.f.s. I=4"/day	Q c.f.s. from unit graph I=4"/day
I	II	III	IV	V	VI	VII
0	0	0		0	0	0
¼	210	105		105	210	
½	400	410		410	820	
¾	572	896		896	1792	
1	748	1556		1556	3112	2200
¼	918	2389		2389	4778	
½	1092	3394		3394	6788	
¾	1278	4579		4579	9158	
2	1442	5939		5939	11878	9400
¼	1600	7460		7460	14920	
½	1740	9130		9130	18260	
¾	1874	10937		10937	21874	
3=D	1986	12867	0	12867	25734	22,280
¼	2080	14900	105	14795	29590	
½	2170	17025	410	16615	33230	
¾	2242	19231	896	18335	36670	
4=tpo	2300	21502	1556	19946	39892	36,680
¼	2128	23716	2389	21327	42654	
½	1900	25730	3394	22336	44672	
¾	1680	27520	4579	22941	45882	
5	1470	29095	5939	23156	46312	46,600
¼	1290	30476	7460	23016	46032	
½	1142	31693	9130	22563	45126	
¾	1018	32773	10937	21836	43672	
6	910	33737	12867	20870	41740	44,120
¼	824	34604	14900	19704	39408	
½	750	35391	17025	18366	36732	
¾	676	36104	19231	16873	33746	
7	582	36733	21502	15231	30462	34,200
¼	542	37295	23716	13579	27158	
½	476	37805	25730	12075	24150	
¾	416	38252	27520	10732	21464	
8	360	38640	29095	9545	19090	21,440
¼	302	38971	30476	8495	16990	
½	240	39242	31693	77549	15098	
¾	184	39454	32773	6681	13362	
9	136	39614	33737	5877	11754	13,400
¼	90	39727	34604	5123	10246	
½	54	39799	35391	4408	8816	
¾	22	39837	36104	3733	7466	
10=too	0	39848	36733	3115	6230	7,480
¼		2553		2553	5106	
½		2043		2043	4086	
¾		1596		1596	3192	

Table 3 (Continued)

11	1208	1208	2416	3,120
[illegible]	877	877	1754	
[illegible]	606	606	1212	
[illegible]	394	394	788	
12	234	234	468	760
[illegible]	121	121	242	
[illegible]	49	49	98	
[illegible]	11	11	22	
13 to	0	0	0	0

come to meet any other drop of water which falls on a point on the former contour at the end of rainfall. Since the intensity of rainfall is constant, the rain falling with this constant intensity at different times within the area bounded by these two time contours contributes a stream flow Q_t at the outlet after an elapse of time, t. We have, therefore,

$$Q_t = \frac{\frac{R}{D}}{12 \times 24 \times 3600} l_{(t-\frac{D}{2})} \times V D$$

$$= \frac{1}{1,036,800} R \cdot l_{(t-\frac{D}{2})} \cdot V \qquad (12)$$

where Q is in c.f.s.; R in inches, $l_{(t-\frac{D}{2})}$ in feet and V in feet per second. $l_{t-\frac{D}{2}}$ is the average length of the time contours and V, the average velocity of flow, in the area bounded by the two time contours.

Now imagine that the duration D is decreasing, and then this area is also reducing as the two time contours t and t-D are approaching each other. In the limit, as D approaches zero, or the rain falls in an infinitesimal time, dt, this long strip of area becomes a line, the length of which is l_t, the average velocity of flow across it is V or $\frac{ds}{dt}$. R becomes dR, while Q, dQ in the sense of differentials. Therefore, dividing each side of the equation by dt, we have,

$$\frac{dQ}{dt} = f(t) = \frac{1}{1,036,800} \frac{dN}{dt} \cdot l_t \cdot \frac{ds}{dt} \tag{13}$$

which is an equation of fundamental importance.

In another form, Equation (13) may be written,

$$\frac{f(t)}{\frac{dN}{dt}} = \frac{1}{1,036,800} l_t \cdot \frac{ds}{dt} \tag{14}$$

Here $\frac{f(t)}{\frac{dN}{dt}}$ is the ordinate of instantaneous hydrograph for unit rainfall intensity of one inch per day at the time t. It is equal to the product of the length of time contour l_t and the average velocity of flow $\frac{ds}{dt}$ through this contour at the same time t, divided by a constant 1,036,800. This is the significance of an instantaneous hydrograph. In other words, the knowing of an instantaneous hydrograph provides a relation between the two very important factors.

Fig. 5 shows time contours and the corresponding instantaneous hydrograph. Section A-A is cut vertically across the time contour along a flow line. The distance between the points O and A on the ground is ds. It takes a time dt for water flowing from A to O. B, C, and D are points in sky. The distances B A, D O and C D are equal to dR. It also takes a time dt for rain falling from B to A, D to O, and C to D. A drop of rainwater at B and another at D will occupy after dt seconds, the positions A and O respectively. They, however, have no chance to meet each other and combine together. A drop of water at C, on the other hand, will combine with that at

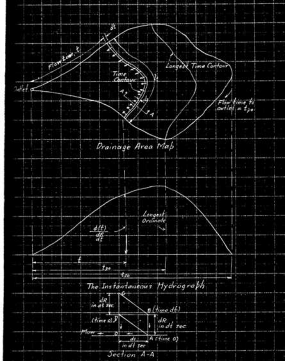

Fig. 5 - The Instantaneous Hydrograph and Time Contours

3 until they arrive at O after 2 dt seconds. So do with the waters at D and A, and all points in between the two verticals CD and BA. Therefore, if we ~~consider~~ the line BA to start with (time 0) and regard those waters in the triangular prism DOA as having already been drained before time = 0, the amount of water that will run together in dt seconds is represented by a long prism, with the cross section ABCD and length l_t. Its volume is $(dR \cdot ds) \times l_t$ combined in dt seconds. Hence the rate of discharge dQ equals $\frac{dR}{dt} \cdot ds \cdot l_t$. Dividing this by dt and introducing the constant $\frac{1}{1,036,800}$ to take care of the units, we also obtain Equation (13). This provides another way of proof.

Equation (14) should be carefully studied and thoroughly understood. For example, for a given t, and hence a given value of $\frac{\frac{dQ}{dt}}{\frac{dR}{dt}}$ from the instantaneous hydrograph, if $\frac{ds}{dt}$ increases due to any reason, (It will be discussed in the theory of varied instantaneous hydrographs, Part IV), Equation (14) would require that l_t must be decreased. This means that the time contour t must be moved forward to or backward from the outlet so as to fit the new l_t. Nevertheless, a judgment by common sense will not approve it. A change of flow conditions at one time contour should not affect those at other places between the time contour and outlet. The value of l_t should not be affected at all. Consequently, if $\frac{ds}{dt}$ changes, it must be $\frac{\frac{dQ}{dt}}{\frac{ds}{dt}}$ that follows its changing. Both l_t and $\frac{ds}{dt}$ are independent variables.

10. Determine Time Contours from an Instantaneous Hydrograph

The Flow Line System - A drop of rainfall, after it is precipitated upon a point in the drainage area, flows overland toward the outlet through trickles, streamlets, tributaries and then main channel. Throughout its trip, there is a definite path of flow from any point to the outlet. This path of flow is known as a flow line.

The law of physical phenomena requires that the flow of water from a point downward follows the direction of the maximum slope. With a contour map given, the system of lines drawn orthogonal at every point to the topographical contours represents the directions of maximum slopes. These lines are, therefore, a set of flow lines.

Interpreted by theories of hydrodynamics, a topographical contour is really an equipotential line. A line joining points of equal elevation, of course means the line joining points of equal potential, inasmuch as gravitation here is the main motive force. A system of topographical contours would, therefore, give a "velocity potential," the divergence of which equated to zero is the Laplace's Equation.

It would seem, then, that the whole problem of the rainfall-runoff correlation could be solved by the methods of hydrodynamics. However, it is not actually possible due to several reasons. (1) The friction to flow plays such an important part that it should not be neglected as it is in a problem of theoretical hydrodynamics. (2) The contours and flow lines are so irregular that any mathematical means of deriving theoretical equations is not possible. (3) The supply of rain water to all points in the area rather than from a single source makes this problem of unsteady flow of extreme complexity such that the mathematical physicists have never considered the possibility of a solution.

The Hydraulics of Overland Flow - The flow of water over the ground surface during rain is so irregular as not to be susceptible of expression in general terms. However, a brief description of the hydraulics of overland flow is instrumental to the method of locating time contours that follows.

Overland flows may be either laminar or turbulent, or the two mixed. Laminar flows occur where there is only a thin film of imperceptible depth while the velocity is extremely slow. Apparently the flow close to the divide of a basin is always laminar. An expression for the velocity of flow can be derived by means of the Poiseuille's law of capillary flow. In the latter, the velocity is directly proportional to the surface slope.

Turbulent flow is what most engineers would imagine as the general type of overland flow. Whenever water attains some appreciable depth on the ground, the flow is probably turbulent. But, it should not be inferred that all flows are turbulent. Surface slopes and roughness may vary locally between wide limits. These variations may bring about a change from one regime to another.

According to the commonly used formulas, the velocity of turbulent flow varies as the square root of the surface slope. The Manning's formula shows that it varies as the two-thirds power of the depth and inversely with the coefficient of roughness of the surface.

Locating Time Contours - Let us integrate Equation (13) between any two contours t and (t-D). Let s_t and s_{t-D} be the corresponding distances of the contours t and (t-D) respectively, which, however, do not represent any actual distance since the distance from a time contour to the outlet is not constant. Thus, we have,

$$\int_0^{Q_t} dQ = \int_{t-D}^{t} f(t)\,dt = \int_{s_{t-D}}^{s_t} \frac{1}{1{,}036{,}800} \frac{dR}{dt} \cdot l_t \cdot ds.$$

~~From Equation (9), $Q_t = \int_{t-D}^{t} f(t)\,dt$, therefore,~~
Or,

$$Q_t = \frac{\frac{dR}{dt}}{1{,}036{,}800} \Big[\text{Area}\Big]_{t-D}^{t} \qquad (15)$$

The latter bracket represents the area included between the two time contours t and t-D.

Evidently, Equation (15) is nothing more than a refinement of Equation (12) from which Equation (13) was derived.

Let us consider that we can always have a rainfall lasting as long as the time of concentration of a given basin. Then we can put t = D in Equation (15).

$$Q_t = \frac{\frac{dR}{dt}}{1{,}036{,}800} \Big[\text{Area}\Big]_0^{t} \qquad (16)$$

which shows that the rate of discharge at the outlet at any time t is proportional to the area included between the time contour t and the outlet, if the rainfall lasts as long as t.

Equations (12), (14) and (16) can be used to determine the time contours of any drainage basin with a given instantaneous hydrograph. The process is as follows:

(1) Obtain a contour map of the area. Starting from the outlet, draw orthogonal lines perpendicular to the topographical contours at every point. Thus obtain the flow lines.

(2) For a preliminary draft of the time contours, locate them roughly by the following means. a. Assume that the trickle and overland flows follow the turbulent regime. Proportion the distances along the flow lines between any two contours inversely as the average slopes. b. For flows in the channels, velocities can be roughly estimated with the knowledge of discharges and stages in the river. Locate the time contours duly. Here, we have the approximate locations of the time contours, but we do not know the time that should be designated to each contour.

(3) Assume a time t, compute Q_t by integrating the instantaneous hydrograph from Equation (2), $Q_t = \int_0^t f(t)\,dt$.

(4) Knowing Q_t, compute $\Big[\text{Area}\Big]_0^t$ from Equation (16). This is the part of the drained area between the outlet and the time contour t.

(5) Find the time contour t on the map that fits approximately this computed area. Thus we designate the time contour.

(6) Measure the length of this time contour, l_t, from the map.

(7) From Equation (14), $\dfrac{f(t)}{\frac{dB}{dt}} = \dfrac{1}{1,036,800}\, l_t \cdot \dfrac{ds}{dt}$, compute $\frac{ds}{dt}$ which is the velocity of flow at the time contour t.

(8) Repeat the steps (3) to (7) for many other time contours. Write down the values of $\frac{ds}{dt}$ on the map.

(9) Knowing the velocities at many points on the map and of course, also the distances between any two points, we can readjust the time contours by shifting the points along the flow lines toward or away from the outlet.

(10) Repeat the above process over and over until it proves satisfactory in step (9).

It should be noted that the above process of determining time contours is a very flexible one. We can locate them as accurate as we need, if we put in enough labor. The more the labor, the more the accuracy we obtain.

The method, in fact, is somewhat a reverse process of the so-called "The Detail Method"[6] as described by R. L. Gregory and C. E. Arnold. In the latter, the writers determine the time of concentration of several separated areas from some estimated velocities and thereby they compute Q from the relation Q = CIA. It seems they do not realize that assuming the velocity of flow is nothing more than assuming the location of a time contour since the distance is definite on a map. Consequently, the whole "Detail Method" is established upon an assumption without theoretical foundation.

11. Contributions of the Time Contour Analysis as Derived from the Theory of Instantaneous Hydrograph.

The theory of instantaneous hydrograph makes possible the

6. "Runoff - Rational Runoff Formulas" Proc. Am. Soc. C.E. April, 1931.

determination of time contours as above described, while the latter on the other hand, offers many important contributions to the science of hydrology and the related subjects. The most important of them are as follows.

(1) The Time Contour Analysis As an instrument to explain the drainage area characteristics. - There are many problems in hydrology that cannot be explained otherwise are readily solved by the instantaneous hydrograph and time contour analysis. For example, engineers usually do not have a picture in their mind of just what makes the peak of a hydrograph. This is readily explained by the time contour analysis. Let the drainage area be divided up by time contours of a constant interval equal to the duration of rainfall. These divided strips of areas are of course not equal in area. It always increases from zero at the outlet to a maximum where the average l_t is longest and then decreases to zero again. For uniform rainfall and a constant $\frac{dH}{dt}$, it is seen from Equation (15) that Q_t is directly proportional to these areas. Consequently, the time of arrival of flow from the largest area with the longest l_t is the time of peak flow. Similarly, other smaller areas cause the minor flows, each arriving at the outlet on the time according to the locations of the respective time contours.

It is very interesting to note that some empirical formulas derived for flood flows contain a factor, "the average width of the drainage basin" such as in the formula published by Major C. E. Pettis.[7] Later, the Committee on Floods of the Boston Society of

7. A New Theory of River Flood Flow, by Charles R. Pettis, Major, Corps of Engineers, U. S. Army, Baltimore, Md., 1927.

Civil Engineers presented an idea quite similar to that contained in the author's Equation (12). The following quotations are made: "...the size of the peak flow depends upon the maximum average width of the water-shed over such a distance as is traversed by the flow during the period of the storm," and "... for an instantaneous storm the peak flow will vary directly with the maximum width." It shows that these ideas are getting nearer and nearer to the truth, except that their mistakes are in considering the controlling factor to be the maximum average width of the drainage area rather than the maximum average length of the time contours, which will, of course, make considerable differences.

After all, the Boston engineers in the Report of the Committee on Floods with A. T. Safford as chairman and C. S. Kent as secretary have done a very brilliant work which is monumental in the history of hydrology. In studying the effects of drainage area characteristics, the report mentions that "the rapidity of the run-off is determined primarily by three features of the drainage area above any point: 1. The magnitude and distribution of the distances and areas above it. 2. The velocity of flow which determines the rapidity with which the run-off reaches it. 3. The amount of pondage or storage available to absorb and retard the natural run-off." As a matter of fact, the instantaneous hydrograph and time contour analysis have automatically taken care of all these three factors at same time.

(2) Applications to non-uniform rainfalls - Once the time contours have been determined, the hydrograph can always be formed

for any given storm no matter how irregularly it is distributed. The method is self-evident, so it will not be discussed here. Moreover, the time contours can be used to determine just how and where an ideal storm should be distributed so as to produce a maximum peak flow. The shape and position of the longest time contours are evidently controlling.

(3) Application to the problems of soil erosion - Nowadays the subject of soil erosion has been received with increasing attention especially in this country. Check dams have been proposed to be built on trickles and streamlets near headwaters. No doubt, in these problems the velocity of overland flow is a major factor. A given velocity of flow determines the maximum size of the soil particles that can be carried by the water. Conversely, a given size of soil has a certain critical velocity to carry it. Knowing time contours, we know the velocity of flow at any point in the basin. For rainfalls of other intensities, the velocities are proportionally changed as shown by Equation (13). (This statement is, however, not true. See discussions in Part IV). Furthermore, the knowledge of the overland velocity is, incidently, a key to the hydraulics of overland flows.

(4) Derivation of hydrographs at any point in the basin - As described before, the drainage system of a basin is a most complex problem of unsteady flow. Along a channel, the discharge increases in point to point from headwater to outlet. Sometimes the hydrograph at the mouth of a tributary or at a point up-stream is required for informations in engineering projects where there is

no gaging station. Sometimes the hydrographs at these places simultaneous to the one measured at the outlet are required. In the latter case, usually the approximate method of routing floods is resorted to. All these problems are, however, solved at once by the time contour analysis. The analysis not only gives hydrographs at the measuring point, but also gives any hydrograph at any point. Moreover, it supplies simultaneous hydrographs at all places. The method, of course, needs no explanation.

In running river models at present time, the sources of water are limited to only one or several places. The discharges of each are simply obtained from rough estimation. However, the experimenters depend upon it for making the verification test, the success of which is considered as the primary proof that the model is correct and serviceable. It would be clear to everyone that how important is the correctness of these values of discharges to the result of experiment. It is also easy to see what effects will result from omitting some sources of flow from tributaries, or from combining them into a single source rather arbitrarily. (As is usually done). As a matter of fact we need to know much about all these, if good results of experiment are desired. The instantaneous hydrograph and time contour analysis tells everything.

One of the mistakes in the modern river model experiments is by running a constant discharge to investigate the effect of a change in the channel such as cut-off, river improvements, etc. The results of experiment are supposed to tell how much the river stages will be raised or lowered at several places. It is so apparent by common sense that the effect of a constant discharge is almost the same as that of the still water if surface slopes are neglected. (except for movable bed experiments.) Then why should we run water through models? It is the effect of variable discharges that we should investigate. A hydrograph rather than a constant flow should be run through the model. The effect of river improvements will then be shown from the modification of a given hydrograph at the upstream end as it is observed at the downstream. The comparison between the peak flows of these hydrographs at the downstream end under different conditions of river improvement will then indicate the amount of raising or lowering. These peaks, evidently, will not happen at the same time. The hydrographs required for investigations, however, can easily be found from the time contour analysis even at places where the river has not been gaged.

(5) A suggested method of finding flood flows for designing culvert capacities and bridge openings where there is no stream measurements at all. - In carrying out her reconstruction program, the Chinese Government has built thousands of miles of railway in recent years way back to the interior where there are no stream measurements at all. The capacities of culverts and bridge openings are roughly estimated by the Talbot's formula as often used in this country in which the climatic conditions are very much different from those of China. Of course, this empirical formula has no theoretical basis, and especially the constant depends so much upon the local climatical conditions. A method has

been proposed to the Chinese Government by the writer which, however, can be equally applied in this country. The outline of the process is as follows:

1. Locate approximately the longest time contours on a map near the place where the width of the area is maximum. (Also see discussions in Section 34). This is done by estimating the approximate velocities of the overland and channel flows during flood time. Supplied with some pictures of channel conditions of known coefficients of roughness, a competent engineer should be able to give a rough estimation by observation in the field. This is also supplemented by the recorded highest marks of flood flows as told by local people.

2. Make a table showing several corresponding values of time and the maximum area as covered by two time contours with this interval of time. Plot it on cross section paper. The curve will naturally rise from t = o.

3. Secure the data showing maximum intensities of rainfall as a function of duration. This may be a general data good for regions in one or two states. It is not necessary to ~~get it~~ be obtained from local rainfall records. Plot it on the same cross section paper. The common base is time. The curve will be asymptotic to the axes on the two positive ends.

4. Multiply the ordinates of the two curves at same times for several points. Plot the products on the same paper, paying due attention to the units. The maximum product is the required peak flow.

The underlying theory will not be treated as it is evident. The idea is to find the maximum value of Q, where $Q = i\,A$, i being decreasing with t, while A increasing with t.

12. On the Unit Graph Method

Recently the unit graph method has been well acknowledged as a device for resolving the complex relations of rainfall and run-off. Unit hydrographs and distribution graphs have been prepared for a considerable number of rivers in the United States as published in the Water Supply Paper 772, "Studies of Relations of Rainfall and Run-off in the United States," by W. H. Hoyt and others.

Defining the theory of the unit graph, LeRoy K. Sherman, the proposer of the theory, states[9] that, "For the same drainage area, there is a definite total flood period corresponding to a given rainfall, and all one-day rainfalls, regardless of intensity will give the same length of base of the hydrograph." "If a given one-day rainfall produces a one inch depth of runoff over the given drainage area, the hydrograph showing the rates at which the runoff occurred can be considered a unit graph for that watershed." He also states that "The ordinates of all graphs of runoff for unit time are directly proportional to the net depths of rainfall in that same unit time. This follows because the bases of the graphs are equal."

Merrill M. Bernard,[10] in 1934, developed certain features of

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9. "Determining Streamflow from Rainfall by Unit Graph Method," Eng. News-Record, April 7, 1932, p. 50.
10. "An Approach to Determine Streamflow," Proc. Am. Soc. C.E., Jan. 1934, p. 5.

the unit hydrograph, introduced added features of the distribution graph and pluviagraph, and suggested certain relations between rainfall and runoff within the storm period.

These graphs are all determined and applied regardless of the duration or actual intensity of rainfall.

Although their methods have met the test of reproducing the stream hydrograph approximately, both Sherman and Bernard have not, however, proved in their papers the two fundamental principles that have been laid as the foundation of the unit graph theory, i.e., a definite total flood period and a definite distribution of runoff, irrespective of the intensity of rainfall. Therefore, these principles can only be taken as assumptions underlying the Sherman's theory.

It is evident that both these assumptions are not true from the theory of instantaneous hydrograph, but are only approximate. First of all, with a given amount of rainfall, say, one inch recorded in one day, the total flood period (and hence the intensity) does depend upon the duration of rainfall, that is, whether the rain falls actually in 24 hours, or in 12 hours, or still less. This is clear when we consider Equation (1),

$$t_c = t_{ce} + D,$$

as given at the beginning of this part, where the writer has mentioned that the instantaneous flood period, too, or the time of concentration, is a constant, irrespective of the rainfall intensity. It follows, therefore, that t_c does depend upon D and hence upon the intensity for a given amount of rainfall.

Next, the ordinates of the unit-graph should not be definite, but should vary according to the duration of rainfall. Studying carefully equations (2) to (6), we will notice that the ordinates of the curves O A and D E on the two ends of the hydrograph determined by the equations $Q = \int_0^t f_1(t)\,dt$, (2), for t equal to O to D, and $Q = \int_{t-D}^{t_{co}} f_2(t)\,dt$, (6), for $t = t_{co}$ to t_o, are definite, while those of the remaining portions of the hydrograph as determined by Equations (3), (4) and (5) for t = D to tco are not, but are functions of the duration D. Because the hydrograph consists of partly constant and partly variable ordinates, it should not be derived by direct proportion as Mr. Sherman does. It is the ordinates of the instantaneous hydrograph that are proportional to the rate of rainfall, but not those of the hydrograph integrated from the former.

The same example for the Big Muddy River at Plumfield, Illinois due to the storm of October 4-6, 1910, has been worked out by the unit graph method, which is listed in Table 3 and plotted in Fig. 4 for comparison. It might seem surprising that the total flood periods figured out from the two methods are just the same. This is because in applying the unit graph method, the total flood period of the unit graph used has been arbitrarily taken as 11 days, whereever it actually should be $10\frac{1}{2}$ days, since there was no rain nor runoff in the first half day when the standard unit graph was determined. (D = 12 hrs.) Consequently, the total flood period as determined by the unit graph method may actually have a maximum possible error of one day's time too long, if the method uses one day as unit.

On account of their equal base, the ordinates of the two hydrographs do not differ from each other very much, the maximum divergence being only 25%.

Besides the above described fallacies of the unit graph method, which, however, will not cause serious errors in the results, there are two other much more essential elements in hydrologic phenomena that the method neglects entirely. These are the problem of water losses (to be described in Part III) and the irregular distribution of rainfalls. (as described in Sections 9 to 11). This is the reason why the method cannot be generally accepted. "As in many other instances in the development of hydraulic science, reliance is placed to the fullest possible extent on available scientific theory, as well as on the cumulative evidence of general relations disclosed by analysis of experience or experiment. This use of these graphs is still largely in the experimental stage, and theoretically and practically there appear to be limitations of application that have not yet been well defined."[11]

11. U. S. Geological Survey, Water Supply Paper no. 772, "Studies of Relations of Rainfall and Runoff in the United States," pp. 123-124.

III - The Rate of Water Losses

13. The Phenomena of Infiltration

Among the several agencies of water losses, infiltration of water into the soils, the rate of which varies both with time and locality, deserves special consideration.

A given soil of definite volume and weight has a definite pore space which may be occupied by air or by water, or shared by both, as the case may be. If the pore space is entirely filled with water, or the soil is saturated, three forms of water are found to be present - hygroscopic, capillary, and free, or gravitational. These forms differ, not in their composition, but in the position that they occupy in relation to the soil particles.

The hygroscopic and capillary water are both film forms; that is, they surround the soil particle, being held partly by the attraction of the particle and partly by the molecular attraction of the liquid for itself. The hygroscopic film is very thin, being water of condensation, or adsorption. When this film is satisfied and moisture is still present, the capillary water film begins to form. The line of demarkation between hygroscopic and capillary water is not sharp. The general difference between the two forms may be considered as being not only one of position, but also one of movement, the power being possessed only by the capillary film. With a change in any controlling condition, such as temperature, hygroscopic may change to capillary, or vice versa. As the capillary water continues to increase and the film becomes thicker and thicker, a point is at last reached at which gravity overcomes the surface tension of the liquid and drops of water form which tend to move downward through the air spaces, being now subject to movement by the attraction of gravity. Free, or gravitational water then also becomes present in the soil. If water is still added, the gravitational water continues to increase until the air is about entirely displaced and a saturated condition results. Similarly, there may be a change of capillary to free water, or vice versa, with a change of structure, temperature, or pressure.[12]

The freedom with which water will move through soils under the action of gravity and capillarity depends upon the factors affecting these two kinds of motions.

The capillary movement may go on in any direction in the soil, since it is largely independent of gravity; yet under natural field conditions the adjustment tends to take place very largely in a vertical direction. When a soil is exposed to evaporation the surface films are thinned and water moves upward to adjust the tension. This explains why such large quantities of soil water may be lost so rapidly from an exposed soil. King shows in the Wisconsin 7th Report, p. 151, the following results from his experiment:

Water evaporated in lbs. daily per square foot.

Distance of W.S. from Ground Surface	1'	2'	3'	4'
Fine sand	2.37	2.07	1.23	0.91
Clay loam	2.05	1.62	1.00	0.90

12. This paragraph is quoted from p. 201, Lyon, Fippin, and Buckman: Soils, Their Properties and Management, 1915.

Capillary adjustment may go on downward, also, as is the case after a shower. Here the rapidity of the adjustment is aided by the weight and movement of the water of percolation. The factors affecting the rate of capillary movement are surface tension, texture and structure of the soil.

Where rainfall is heavy enough, gravitational flow takes place through the pore space proper, and also through the cracks, the worm tracks, the passages left by the decayed roots and other adventitious openings in the soil. The percolation proceeds until the zone is reached where the pore space is completely filled. This is the water table. The factors affecting the movement of gravity flow are temperature, pressure head and texture and structure of soil.

The surface runoff may range all the way from nothing to almost totality, according to the nature of the soil and the condition of its surface. Sandy soils, especially when coarse, may absorb instantly even a very heavy rainfall. Heavy clay soils when dry will at first also absorb quickly quite a heavy precipitation. However, as the beating of the raindrops compacts the surface, the absorption quickly slows down, so that heavy downpours of brief duration, while wetting thoroughly into a plastic mass in the first two or three inches of a clay soil, may leave all beneath dry, to be very gradually moistened by the slow downward percolation against the resistance of the air in the soil; while the greater part of the later portion of the shower will drain off the surface in muddy runlets. This shows a possibility of hundred per cent runoff.

It will also be noticed that there is a great diminution in the rate of flow when a soil contains small clay particles in the naturally puddled condition. The particles constituting the clay are no longer aggregated, and their colloidal "gel" coating absorbs water and swells, whereby pore spaces between them must become extremely small. Not only is the flow diminished by the increase of friction in the narrow channels, but in the case of clay their dimensions have become so small that probably the contained water is wholly within the range of molecular forces.[13] It is thus prevented from flowing at all, and only moves by diffusion.

14. The Rate of Infiltration - Results of Experiment

As previously described, the downward movement of water through soil is due to gravity and the downward pull of capillary forces. At different locations in a drainage area, the kind of soil may change from place to place. The rate of infiltration is dependent primarily on the texture of the soil, its structure including shrinkage cracks and channels formed by decayed roots and burrowing animals, and is affected by the character of salts in the soil solution and the temperature. Furthermore, this rate changes as time goes on since the beginning of rainfall. Fig. 8 shows the rates of capillary rise and of percolation for several materials. The capillary rise is against gravity while the downward percolation is accelerated by gravity. Both are presented here

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13. The Soil, by Sir A. D. Hall, p. 89.

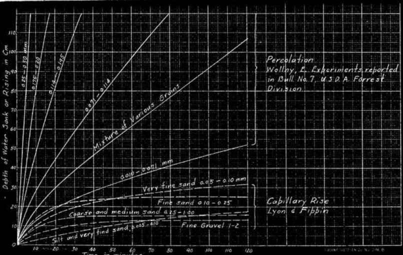

Fig. 6 – Rates of Percolation and Capillary Flows

for comparison.

Wollny's experiments[14] of percolation were made with soil of varying grain, placed in tubes 110 cm. deep, the water dropping constantly on top of the soil column. His results gave the following conclusions:

(1) Water is conducted downward more rapidly the larger the soil particles (i.e., the less capillary attraction exists).

(2) The non-capillary interstices of the soil accelerate the downward movement of the water. (i.e., the less mechanical obstruction of the soil particles).

(3) The granular soil, the water penetrates faster than in powdery soil (i.e., penetration is the slower the denser the stratification). It is most rapid in quartz and slowest in clay; in humus at a rate between these two, but in a mixture of clay soil and humus faster than the average of the two.

(4) The rapidity of drainage in a granular soil is independent of the size of the grain.

As a recapitulation of the foregoing discussions, the following conclusions on the rate of infiltration are deducted:

(1) Beginning at the instant the rain touches the ground, the rate of infiltration, in general, decreases from a maximum generally to a more or less constant value as a result of the combined action of capillary and gravitational flow. (See Fig. 5).

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14. Untersuchungen über Kapillare Leitung des Wassers im Boden. Forsch. a.d. Gebiete d. Agri-Physik, Band 6, 7, and 8, 1884.

The time required before maintaining a constant rate depends upon the sizes of grain, the smaller sizes taking a shorter time.

(2) Notwithstanding that the surface layer of the ground may possibly be saturated by a heavy rainfall, this zone of saturation descends all the time followed by the percolating water. Therefore, there is rarely a case of hundred per cent runoff except under one of the following conditions: a. The water table is so high as almost up to the ground surface. b. In the case of heavy clay soils, the beating of rain drops compacts the surface, thereby forming a temporary impervious plastic mass at the first two or three inches during the raining period. c. For clays containing colloidal matter, it absorbs water and swells, leaving the pore spaces so small that the contained water is within the range of molecular forces, thereby stopping the percolation.

(3) The rate of percolation is so slow (See Fig. 6) that unless the water table is quite near the ground surface, (4 feet or less) for most of the time the percolating water will not reach the water table before a continuous rain ceases. Therefore, the initial elevation of water table is not important in the role of infiltration.

15. The Observed Water Losses

It is evident that the surface runoff from a drainage basin is a residue of the rainfall after all water losses have been deducted. The ordinary expression of runoff in percentages of rainfall, however, has no significance at all. A different amount of rainfall will not proportionally change the amount of runoff.

The amount of water losses varies with the season of the year, as the temperature and ground conditions are different. The rate of water losses in July is, obviously, much greater than that in January. Suppose that the cycle of the physical phenomena on the earth coincides with the cycle of the calender year, then the temperature and ground conditions in a given drainage area would be exactly the same on the same day of different calender years. Consequently, the rate of water losses would also be invariable on the same days, and correspondingly, there would be a definite curve of the rate of water losses against time, such as shown in Fig. 6.

In case the intensities of rainfall are high enough to supply the required rate of water losses at all times and in all places of the basin, then the amount of water losses will evidently be the same for any rainfall of a same duration. The rate of runoff in any point at any time during the storm period will be the difference between the rainfall intensity and the required rate of water losses at that time.

These considerations led the writer to have the idea that, if a negative hydrograph at the outlet can be constructed to represent the flows that would exist in addition to the observed flows if there were no water losses, such a negative hydrograph will be definite for any rainfall under the prescribed conditions. The total amount of water losses as found from the difference between the total precipitation and total surface runoff for several storms may serve as a test of this theory.

Tables 4 and 5 give the calculations of surface runoff due to two storms over the Big Muddy River Basin above Plumfield, Illinois. These are to be compared with Table 1 of section 6. The storm in Table 1 occurred in March 6, 1926; that in Table 2 in March 18, 1926; and that in Table 3 in April 6, 1928. The first two storms were only 12 days apart in the calender years, so that we would expect that the temperatures and ground conditions do not differ from each other very much. The last storm, in April 6, was however, a month or 31 days from the storm of Table 1, i.e., March 6, so that we would expect that the amount of water losses should be greater in April than in March.

All these three rainfalls were distributed quite uniformly over the basin, although that of March 18, 1926 was not so ideal as the other two. It should be noted, however, that the amount of water losses does not depend in any way, upon the uniformity of storm distribution, so long as the rainfall at any time and at any place in the basin is sufficient to supply the required water losses.

The results from these tables show that from the storm of March 6 the water losses were 0.51", from the storm of March 18, 0.50", which are remarkably close to each other. The storm of April 6 contributed 0.62" for the water losses, its greater amount being duly expected. Incidently, these tables were originally constructed for the purpose of studying the distribution of flows (Column V) rather than for investigating the amount of water losses.

Table 4 - Calculations of Surface Runoff

(Runoff records from U. S. Water Supply Paper No. 605)

I	II	III	IV	V	VI	VII	VIII	IX
Date 1925	Observed Runoff c.f.s.	Depletion Flow c.f.s.	Net Runoff c.f.s.	Discharge for 1" run-off c.f.s.	Day	Rainfall, in. Mt. Vernon 43%	Benton 57%	Wt. Av. 100%
Mar. 11	51	51				.17	.36	
12	112	112						
13	78	78						
14	351	351				.32	.65	
15	583	583						
16	538	538						
17	377	377	0	0	0			
18	368	288	80	250	1	.90	.76	.82
19	1160	240	920	2900	2			
20	1520	200	1320	4170	3			
21	1670	170	1500	4730	4			
22	1460	150	1310	4130	5			
23	906	136	770	2430	6			
24	463	123	340	1070	7			
25	265	140	130	410	8	.38	.19	.27
26	230	170	60	1920	9			
27	219	219	6430	20280 c.f.s./day				
28	219	219	26.9 / 239 in. - mi.					

Runoff = $\frac{239}{753}$ = 0.317"

Rainfall = 0.82"

Water losses = 0.50"

Table 5 - Calculations of Surface Runoff

(Runoff Records from U.S. Water Supply Paper No. 665)

I	II	III	IV	V	VI	VII	VIII	IX
Date 1928	Observed Runoff c.f.s.	Depletion Flow c.f.s.	Net Runoff c.f.s.	Discharge for 1" run-off c.f.s.	Day	Rainfall, inches Mt. Vernon 43%	Benton 57%	Av. 100%
Apr. 5	77	77	0	0	0	.12		.05
6	374	104	270	1030	1	.86	.89	.88
7	880	110	770	2930	2	.08	T	.04
8	1060	130	930	3530	3			
9	1140	140	1000	3800	4			
10	1160	160	1000	3800	5			
11	1000	170	830	3150	6			
12	584	160	420	1600	7	.31		.13
13	231	140	90	340	88		.30	.17
14	144	134	10	40	9			
15	136	136	0	0	10			
16	185	185	5320	20220 c.f.s./day				

$\frac{26.9}{198}$ in.-mi.

Runoff = $\frac{198}{753}$ = 0.263"

Rainfall = 0.88"

Water Losses = 0.62"

16. A Method of Determining the Amount of Water Losses

The theories and methods as given in Part II: Drainage Area Characteristics, are based upon three assumptions: (1) A uniform rainfall covers the entire drainage basin. (2) There is no water lost throughout its trip to the outlet so that all rainwater is drained from the surface of the watershed, and therefore, the rate of rainfall excess is equal to the intensity of rainfall. (3) The ground conditions are invariable with the rising of river stages so that there is only one instantaneous hydrograph during any storm period. All of these three conditions, however, can hardly be observed in nature. Therefore, methods have to be devised to take each of them into account. The present part is devoted to the correction of the assumed condition (2). The rest will be treated in Part IV.

Let us take two uniform rainfalls, first assuming an equal duration. These two rainfalls are selected under those conditions as prescribed in the above section, i.e., that they occurred in about the same time of two years so that the ground conditions were about the same. The intensities I_1 and I_2 are plotted in Fig. 7, which are invariable with either time t or the x, y coordinates in the area. The rate of water losses L which is the same for both storms, however, varies irregularly with the three variables x, y and t. Let the observed hydrographs be represented as shown in the figure. Evidently, Hydrograph 1 is due to the varying rainfall excess 1, and Hydrograph 2 due to rainfall excess 2. Just because this source of the surface runoff supply (i.e., the

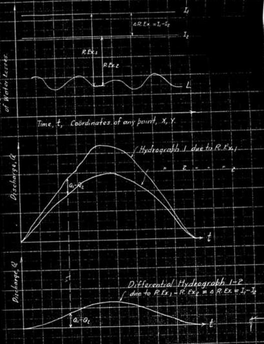
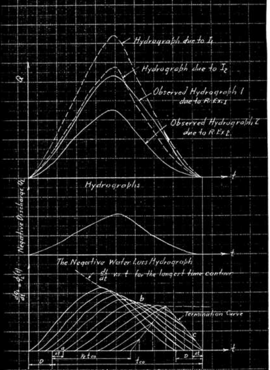


Fig. 7 The Differential Hydrograph.

rainfall excess) is not uniform at all with respect to x, y or t, any analysis applied to the observed hydrograph, such as that described in Part II or the unit-hydrograph method which are all based upon a constant rainfall excess assumption (equivalent to uniform rainfall with no water losses.) is to no avail. Now suppose we plot the difference of the two hydrographs as shown in Fig. 7. This Differential Hydrograph 1-2, is due to the difference of the two rainfall excesses, or R. Ex. 1 - R. Ex.2. But the latter, as seen from the upper diagram, is equal to △R. Ex.1 or $I_1 - I_2$. Thereby, the irregular rate of water losses that can hardly be analyzed by the elaboration of human efforts is automatically eliminated in this simple process.

As this differential hydrograph 1-2 is due to a uniform differential rainfall excess $I_1 - I_2$, we can then treat it by the theoryes of instantaneous hydrograph. An instantaneous hydrograph so derived is good for all cases of applications, not only for the date of the year on which it was derived.

Next, the ordinates of the differential hydrograph 1-2 are multiplied by a factor, $\frac{I_1}{I_1-I_2}$, the resulting hydrograph being one due to a rainfall I_1 with no water losses. This is shown dotted in Fig. 8. Similarly, if the ordinates of the differential hydrograph are multiplied by the factor $\frac{I_2}{I_1-I_2}$, the resulting hydrograph is the one due to the rainfall I_2 with no water losses. These latter hydrographs after subtracted by their respective observed hydrographs should give a common "negative water loss hydrographs" as the one given in Fig. 8.



The Instantaneous Hydrograph for Water Losses

Fig. 8 Water Losses Diagram

The negative water loss hydrograph so obtained represents the flows at the outlet that would exist in addition to the observed flows if there were no water losses. It can also be analyzed into instantaneous hydrographs. The latter, of course, will have the same set or sets of time contours derived from the differential hydrographs. However, the constituent instantaneous hydrographs of water losses are different from those of the differential hydrograph. If the lower diagram in Fig. 8 is compared with Fig. 1, Part II, it will be found that the number of instantaneous hydrographs in the former is much greater than that of the latter. In the latter, (Fig. 1) the number is limited within the duration of rainfall, because there is no rain falling after the end of duration. In the former, (Fig. 8) however, there are instantaneous hydrographs after the end of duration, because then the agencies of water losses are still active although the rain has already ceased.

If the rate of water losses is uniform in the area, the instantaneous hydrographs for water losses within the storm period D are, evidently, similar to those from the differential hydrograph, with ordinates proportional at the respective times. After that, each instantaneous hydrograph is ended at a termination curve as shown in Fig. 8, before it extends to the t - axis. This curve is determined as follows. Take the instantaneous hydrograph abc for example. It occurred at the time (D+Δt) after the beginning of rainfall, or Δt after the end of rainfall. At that time, the last drops of rain falling on the remotest time contour, too, has advanced to the time contour (too - Δt). Between these two time contours there has been left no single drop of water at all. Consequently, there is no water that can be lost by any means. This explains why the instantaneous hydrograph abc is ended at o, Δt from the end of the hydrograph. By analogy, it can be generalized to a rule that for any instantaneous hydrograph with its initial point on t - axis at a time t from the end of duration D, it must be ended also at a point with time t from the end of hydrograph. Thereby, the termination curve can be fully determined. It follows then, that there is no more new instantaneous hydrograph to start with beginning from a point $\frac{1}{2}$ too after the end of rainfall duration.

It should also be noted that the ordinates of instantaneous hydrographs for water losses decrease as time goes on, since the rate of water losses decreases with time. (Section 14) Fig. 8 shows one curve drawn for the longest time contour. In case the general relation between the rate of water losses and time is known, then the equation of each instantaneous hydrograph can be formulated by mathematical means. Generally, the rate of water losses maintains a quite constant value in a reasonable short period (See Fig. 6) especially for heavy rainfalls on pervious soils. The end of this short period may even occur before active runoff begins. Therefore, if we put the origin of the hydrograph at the time when active runoff begins, (strictly, this should always be done,) a constant rate of water losses can reasonably be assumed, whereby the instantaneous hydrograph can be duly determined.

Thus, for $t = o$ to $(D + \frac{tco}{2})$,

$$Q_L = \frac{1}{1,036,800} \int_o^t \frac{dL}{dt} \cdot l_t \cdot ds \qquad (17)$$

Differentiating, we have,

$$f_L(t) = \frac{1}{1,036,800} \frac{dL}{dt} \cdot l_t \cdot \frac{ds}{dt}. \qquad (18)$$

in which $\frac{dL}{dt}$ represents the average rate of water losses along the time contour t.

To find the observed hydrograph, the following equations are useful: When $o < t < D$,

$$Q_t = \int_o^t f_R(t)\, dt - \int_o^t f_L(t)\, dt$$
$$= \int_o^t \left[f_R(t) - f_L(t) \right] dt \qquad (19)$$

When $D < t < tco$,

$$Q_t = \int_{t-D}^t f_R(t)\, dt - \int_o^t f_L(t)\, dt.$$
$$= \frac{1}{1,036,800} \int_o^t \frac{dR}{dt}\, l_t\, ds - \frac{1}{1,036,800} \int_o^{t-D} \frac{dR}{dt}\, l_t\, ds$$
$$- \frac{1}{1,036,800} \int_o^t \frac{dL}{dt}\, l_t\, ds \qquad (20)$$

Differentiating Equation (20), we have,

$$\frac{dQ}{dt} = f_R(t) - f_R(t-D) - f_L(t) \qquad (21)$$

If the two rainfalls used for the differential hydrograph are of different durations then,

$$f_{\Delta R}(t) = \left(\frac{dQ}{dt_1} - \frac{dQ}{dt_2}\right) + \left[f_{\Delta R}(t-D_1) - f_{\Delta R}(t-D_2) \right] \qquad (22)$$

The method of determining and eliminating water losses suggested is a very useful tool in solving hydrologic problems. It can be equally applied even if the unit-graph method is resorted to. The underlying theory is perfectly sound under the assumed conditions of selecting data. If the data are accurate to 100%, the method will yield results of 100% accuracy also. However, there is one point of deficiency that might be seriously criticized. The primary assumption of the method is that the rainfalls are heavy enough to supply the required rate of water losses at all places and at all times. This condition can hardly be realized in an actual example at the beginning of rainfall. Hence, the absorptive power of the ground is so high that it takes all rain that drops on it. Then, the amount of water losses in the two rainfalls will not be the same, the heavier rainfall always losing more than the lighter. Nevertheless, this deficiency can be eliminated if the origin of the hydrograph is begun at the time when acting runoff begins.

For non-uniform rainfalls, the method still holds but will be modified by the analysis given in Part IV.

Usually a greater part of the amount of water lost in a storm (equal to the total precipitation minus total runoff) is attributed to the infiltration and detention before active runoff begins, an accurate determination of which is, therefore, more important than that of the negative water loss hydrograph. This initial loss due to infiltration, however, depends mainly upon the initial moisture deficiency of the soil. An immediately precedent rainfall evidently causes a lower initial moisture deficiency. There is an entirely definite initial moisture deficiency to begin the active runoff for a given intensity of rainfall, which can be determined by the method to be shown in Section 27. As to the actual initial moisture deficiency at the beginning of a rainfall, with the precedent rainfalls given, it can only be found by working on many examples in analysis from the definite initial moisture deficiency subtracted by the known initial water losses.

IV - An Exact Analysis

17. The Fallacy of the Constant Flood Period Hypothesis

It has been shown in Section 8, Part II, that the total flood period (t_c) is not a constant for a given point on a stream from considerations of the theory of instantaneous hydrograph. This is because

$$t_c = t_{co} + D \qquad (1)$$

in which the writer claimed that the instantaneous flood period t_{co}, or the time of concentration is a constant, irrespective of the rainfall intensity. It will now be shown that even the latter, t_{co}, is not constant for different storms.

The constant flood period hypothesis was first established by the late Emil Kuichling who found by plotting many flood hydrographs of the same basin that the hypothesis is nearly true. Later, in September, 1930, the Boston Society of Civil Engineers in the Report of the Committee on Floods employed this hypothesis to derive flood formulas from hydrographs. In this report, the correctness of the hypothesis has been discussed, mainly directed, however, toward the effect of duration as given in Equation (1). For instance, it is stated on page 383 of the report that, "... the total flood period in a given stream, in so far as possible, should be based upon a storm approaching its concentration period," from which it seems that the Boston engineers had not realized, that there can hardly be a rainfall lasting as long as the concentration period. Besides, the hypothesis also forms one of the fundamental assumptions of the Sherman's unit-graph method, or the Bernard's distribution-graph method. In short, it has been widely accepted to the extent that its correctness has rarely, if ever, been studied.

The incorrectness of the hypothesis may be explained as follows. As time goes on from the beginning of rain, the stage of river gradually raises. The results of stream gaging measurements show that the average velocity of flow, in general, increases with stage unless there is a sudden widening of channel section such as from a low water channel to a flood channel. Thus, the average velocity of the Mississippi River at Vicksburg, Mississippi is 1.5 or 2 feet per second at low waters and 6 or 8 feet per second at high waters. It follows then that the surface velocity also increases with stage, since it bears a quite definite relation with the average velocity. When a drop of water drains to a stream channel at a certain stage, it is given a definite velocity at that stage. At another stage it is given another velocity. Consequently, time contours are different at different stages. At high stages, the time of concentration or the instantaneous flood period, too, is shorter than that at low stages.

It is obvious then, that the instantaneous hydrograph also changes with river stages. It is not at all constant or definite for different storms or in the same storm at different stages. In the same degree, a set of time contours is not constant for all stages. The method of finding the instantaneous hydrograph in Part II is, however, simply presented for convenience of explanation. It is based upon the assumption that the ground conditions (including depth of water on the ground and river stages) are constant.

The question naturally arises why the total flood period, tg. does sometimes prove to be quite definite as many engineers have found in working on various hydrographs. The disproof by the above theoretical analysis would not seem strong enough to overthrew the practical results. The reason is this: in working on a flood hydrograph, after the depletion flows have been deducted, the net runoff hydrograph usually shows, on its recession side, a quick drop to a low discharge and thencefrom a very slow, gradual decrease, asymptotic to the depletion curve. A person usually cuts the hydrograph at this end of the steep slope and ignores the gradually decreasing part. Such a process makes the total flood period rather uniform for different hydrographs. Another fact is that hydrographs with about the same initial stages and subsiding to approximately the same final stages do give a nearly definite total flood period. Hydrographs constructed on this basis evidently have nearly constant bases.

However, the total flood periods for different storms do not actually vary greatly even when the recession curve is fully produced. A difference of one or two days would be considered as great. The explanation is found in the following. The part of recession curve after the surface and trickle flows has ended, represents the hydrograph of draining water from the stream channel. Such a curve has a definite shape independent of the intensity or duration of storm, and yet it constitutes the longest part of the whole hydrograph. The rest or the beginning part of the hydrograph representing flows when surface and trickle flows are active, does not occupy much time, although it varies much with different storms. Consequently, for the two parts added together, the total flood period will be more or less constant.

18. The Theory of Variant Instantaneous Hydrographs

Consider a uniform rainfall excess of unit rate, say one inch per day, covering the entire drainage area, and lasting a duration D. Let us divide this duration into an infinite number of infinitesimal times, dt. Corresponding to each instant, there is an instantaneous hydrograph, which varies at different instants of time. Nevertheless, no matter how it varies in its shape, the area included above the time-axis must be a constant, which represents the unit rate of rainfall excess.

Fig. 9 shows a diagram of the variant instantaneous hydrographs. The initial one at t_{01} has the longest base, t_{1co}, but the lowest peak. The final one at $t = D$ has the shortest base but highest peak. The areas under each of them are, however, equal. Suppose the drainage area is divided by time contours as sketched. Let t_1, t_2, etc., represent the times corresponding to the discharges Q_1, Q_2, etc., of the observed hydrograph. The time contours designated by this set of times represent the resultant time contours under the conditions of flow from the initial stage up to the stages as indicated by the respective times. Thus at t_1, the observed hydrograph indicates Q_1 which is equal to the area included between the time contour t, and the outlet multiplied by the rate of rainfall excess. Corresponding to this interval of

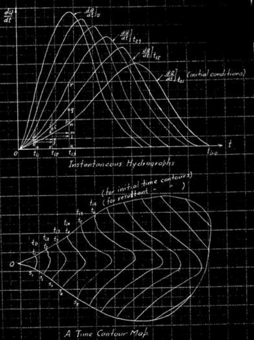


Fig. 9 Variant Instantaneous Hydrographs

time from o to t_1 or t_{01}, an instantaneous hydrograph is shown. Similarly, the instantaneous hydrograph at t_{12} seconds after the beginning of rainfall (taking t_{12} as the average of t_1 and t_2) corresponds to the conditions of flow at the times from t_1 to t_2.

Next let t_{11}, t_{22}, etc., designate the times of the same set of time contours but under the initial conditions of flow. In other words, the distance to the outlet from the time contour t_{11} under the initial conditions is equal to that from the time contour t_1 under the resultant conditions of flow, and is equal to s_1. Similarly, the statement holds true for other times. It is evident, of course, that $t_{11} > t_1$, $t_{12} > t_2$, etc.

We have, then, for discharges within the storm period,

$$
\begin{aligned}
Q_0 &= 0 \\
Q_1 &= \int_0^{t_{11}} \frac{dQ}{dt}\Big]_{t_{01}} \cdot dt = \int_0^{S_1} \frac{1}{1,036,800}\, i_{t_{01}}\, ds \\
Q_2 &= \int_{t_{11}}^{t_{12}} \frac{dQ}{dt}\Big]_{t_{01}} dt + \int_0^{t_{11}} \frac{dQ}{dt}\Big]_{t_{12}} dt \\
&= \int_{S_1}^{S_2} \frac{1}{1,036,800}\, i_{t_{01}} \cdot ds + \int_0^{S_1} \frac{1}{1,036,800}\, i_{t_{12}} \cdot ds \qquad (23) \\
Q_3 &= \int_{t_{12}}^{t_{13}} \frac{dQ}{dt}\Big]_{t_{01}} \cdot dt + \int_{t_{11}}^{t_{12}} \frac{dQ}{dt}\Big]_{t_{12}} \cdot dt + \int_0^{t_{11}} \frac{dQ}{dt}\Big]_{t_{23}} \cdot dt \\
&= \underbrace{\int_{S_2}^{S_3} \frac{1}{1,036,800}\, i_{t_{01}} \cdot ds}_{} + \underbrace{\int_{S_1}^{S_2} \frac{1}{1,036,800}\, i_{t_{12}} \cdot ds}_{} + \underbrace{\int_0^{S_1} \frac{1}{1,036,800}\, i_{t_{23}}}_{}
\end{aligned}
$$

Discharges from: the 1st interval, the 2nd interval, the 3rd interval.

$Q_4 =$ ....
....

In the equation for Q_3, the area contributing the Q_3 has been divided into three parts by the time contours, t_1, t_2 and t_3. The confluence of the flowing waters together to form Q_3 at the end of t_3 seconds, requires that the rains on the area between the contours t_2 and t_3 must fall during the first time interval 0 to t_1, that between the contours t_1 and t_2 during the second interval t_1 to t_2, and that between the contour t_1 and the outlet during the third interval t_2 to t_3. (Refer Section 5, Part II). As shown in Fig. 9, for the first interval o to t_1 the instantaneous hydrograph is $\left.\frac{dQ}{dt}\right]_{t_{01}}$, for the second interval t_1 to t_2 it is $\left.\frac{dQ}{dt}\right]_{t_{12}}$, for the third interval t_2 to t_3, it is $\left.\frac{dQ}{dt}\right]_{t_{23}}$. Therefore, Equation (23) results.

For discharges after the rainfall has ended, we have the following set of equations:

$$Q_{D+1} = \int_{t_{1D}}^{t_{1(D+1)}} \left.\frac{dQ}{dt}\right]_{t_{01}} \cdot dt + \int_{t_{1(D-1)}}^{t_{1D}} \left.\frac{dQ}{dt}\right]_{t_{12}} \cdot dt + \cdots + \int_{t_{11}}^{t_{12}} \left.\frac{dQ}{dt}\right]_{t_D} \cdot dt$$

$$Q_{D+r} = \int_{t_{1(D+r-1)}}^{t_{1(D+r)}} \left.\frac{dQ}{dt}\right]_{t_{01}} \cdot dt + \int_{t_{1(D+r-2)}}^{t_{1(D+r-1)}} \left.\frac{dQ}{dt}\right]_{t_{13}} \cdot dt + \cdots + \int_{t_{1r}}^{t_{1(r+1)}} \left.\frac{dQ}{dt}\right]_{t_D} \cdot dt \qquad (24)$$

in which the first term is the discharges from the first time interval o to t_1, the second term those from the second interval t_1 to t_2, and the last term those from the last interval at the end of duration.

The diagram in Fig. 9 explains the terms of Q_1, Q_2 and Q_3 of Equations (23) geometrically. Thus, Q_1 is represented by the ordinate ab on the instantaneous hydrograph $\left.\frac{dQ}{dt}\right]_{t_{o1}}$. The limits of integration in Equation (23) for Q_1 must be o and t_{11} rather than o and t_1, because the former designates the times of the initial time contours which are derived from the instantaneous hydrograph $\left.\frac{dQ}{dt}\right]_{t_{o1}}$. Q_2 is represented by the ordinate $\overline{e\ i}$ which is composed of two parts: $\overline{e\ g} = \overline{a\ c}$ denoting the first term of the equation for Q_2, and $\overline{g\ i} = \overline{f\ h}$ denoting the second term of the equation for Q_2. The limits of integration t_{11} and t_{12} correspond to the initial conditions t_{o1} in order to integrate $\left.\frac{dQ}{dt}\right]_{t_{o1}}$. Strictly speaking, the limits for the second term should correspond to the conditions of flow for the second interval so as to integrate $\left.\frac{dQ}{dt}\right]_{t_{12}}$. A new system of symbols for t's has to be introduced. But in view of the confusion of twoo many symbols and the fact that the upper limit of the second term must be equal to the lower limit of the first term, t_{11} is still used. Similarly, Q_3 is represented by the ordinate $\overline{K\ Q}$, composed of three parts: $\overline{l\ k} = \overline{a\ d}$, $\overline{l\ p} = \overline{g\ j}$, and $\overline{p\ q} = \overline{m\ n}$, denoting respectively the first, second, and third terms of the equation for Q_3 in (23). The diagram reveals manifestly the fact that the instantaneous hydrograph found from differentiating an observed hydrograph always falls at all ordinates between the instantaneous hydrograph at t_{o1}, and that at the time of the particular ordinate. Thus, the point q at t_{13} is between n and h, i at t_{12} between h and j, and c at t_{11} is of course, on c itself.

At first glance it would appear that Equations 23 and 24 could not be solved since all the $\frac{dQ}{dt}$'s are unknown and also the limits of integration t_i's. But by means of the theory of limits, making each separate divisions of t_{i1}, t_{i2}, t_{i3}, etc. approach zero, and hence also os_1, s_1s_2, s_2s_3, etc. approaching zero, we have,

$$Q_3 = \int_0^{s_3} \frac{1}{1{,}036{,}800} \; 1_{t_0}\, ds - \int_0^{s_2} \frac{1}{1{,}036{,}800} \; 1_{t_1}\, ds$$

$$+ \int_0^{s_2} \frac{1}{1{,}036{,}800} \; 1_{t_1}\, ds - \int_0^{s_1} \frac{1}{1{,}036{,}800} \; 1_{t_2}\, ds$$

$$+ \int_0^{s_1} \frac{1}{1{,}036{,}800} \; 1_{t_2}\, ds - \int_0^{0} \frac{1}{1{,}036{,}800} \; 1_{t_3}\, ds$$

$$= \int_0^{s_3} \frac{1}{1{,}036{,}800} \; 1_{t_0}\, ds$$

$$= \int_0^{t_{i3}} \left.\frac{dQ}{dt}\right]_{t_0} dt$$

$$= Q\,(t_{i3})$$

Therefore,

$$Q_r = Q(t_{ir}) \tag{25}$$

in which Q denotes discharges as a function of t_i.

Similarly, to solve Equations (24) it can be shown that,

$$Q_{D\,r} = Q\,(t_{i(D+r)}) - Q(t_{ir}) \tag{26}$$

The above equations give the following results: (1) Only the set of time contours under the initial conditions, as t_{i0}, t_{i1}, t_{i2}, ... etc. that enters into the function are needed. (2) The equations give a set of data expressing t against $Q(t_i)$ as derived from the observed hydrograph. It should be noticed that t and ti are different as mentioned previously. Therefore, the function Q, and hence $\frac{dQ}{dt}$, are still unknown. A method of solution will be given in the next section.

19. An Approximate Method of Solution for Synthesis

The results of Equations (25) and (26) show that the functions

Q and $\frac{dQ}{dt}$ are still unknown so that none of the instantaneous hydrographs can be found, nor any set of the variant time contours can be located. The determination of these functions requires the knowing of the relation between t, the resultant time contours, and t_1, the initial time contours; both of which are, however, equal in the actual distance laid on the map from the time contour to the outlet, as they represent the same time contour. (See Fig. 9) The writer has not hitherto been able to device a method for determining the individual instantaneous hydrographs. It is obvious that even if this could be done, the results would be so complicated as to be deprived from practical applications.

Notwithstanding the complexity of a precise solution, a method has been devised for practical purposes. The Equations (25) and (26) give a set of data expressing t against Q (t_1) from the observed hydrograph. Let us determine the instaneous hydrograph from them by the method described in section 6 of Part II. Then, from these two sets of data, the set of time contours representing the resultant flow conditions can be located on the map according to the process as described in section 10 of Part II.

It will be shown later in Part V that a method can be devised to determine the relation between the discharges, Q, and the corresponding storages of water, V, on the drainage area. Such a relation is definite for all approximately uniform rainfalls on the area. This is because for these rainfalls, a given discharge Q has definite stages at the outlet as well as at other

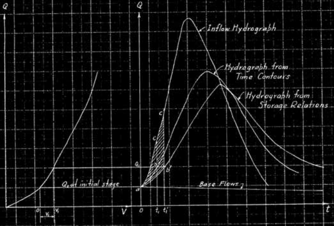


Fig. 10 Correction of Hydrograph in Synthesis

places in the area, so that the amount of water stored in the area, V, is also definite. Let this relation be determined as shown in Fig. 10.

In the process of synthesis, it is required to determine the hydrograph resulting from a given rainfall. The whole difficulty here is that the flow conditions for the present rainfall are different from those during the rainfall used in analysis, mainly due to the different stages of water in the channels. (The ground conditions are the same when rainfalls are taken from the same month of the calender year.) Suppose we obscure this fact for a moment, and find out the hydrograph accordingly by using the known instantaneous hydrograph and time contours. The hydrograph so obtained (Fig. 10) shall be subjected to corrections.

If the given rainfall is not very distortedly distributed, evidently its Q-V relation should be the same as that which has been determined. Fig. 10 shows the method of correction. First, the base flow curve is drawn from the initial flow Q_0 at the stage a. Upon it plot the inflow hydrograph of the given rainfall which is found as follows: An inflow hydrograph of the rainfall used for analysis should be previously obtained by the methods to be described in Sections 25 or 26. It represents the hydrograph of flows into the stream channel. However, such a hydrograph can hardly be analyzed into instantaneous hydrographs. The new inflow hydrograph for synthesis due to a different rainfall duration can only be obtained from it by an approximate method similar to that used in the Unit-graph method. Then for any point b on the hydrograph from time contours, the area abc (bc a vertical ordinate) represents the amount of water stored above the initial stage a up to the stage at b. This should be equal to V_1 as indicated by the storage curve. If actual measurement shows not, a point b^1 is to be determined such that the area ab^1c^1 equals V_1. This point b^1 is on the corrected hydrograph. The latter can thus be determined by trials for successive increments.

If the new time t_1 corresponding to V_1 at b1 is located back on the time contour map and similarly for other times, a new set of time contours is obtained which represents the conditions of flow on the area for the given rainfall in synthesis. Of course this step is not necessary in practice.

In fact, the corrected hydrograph has been obtained entirely from channel storage relations. Its accuracy depends solely upon the accuracy in which the inflow hydrograph is determined. The writer would not claim that the method is a good one, although it is herewith presented.

As discussed at the end of Section 17, most part of the recession hydrograph has a definite shape independent of the intensity or duration of a storm. It is the first part before peak flows that varies much with different flow conditions. However, this part usually lasts only short intervals among the total flood period. Therefore, in an actual problem, the correction of a hydrograph determined from time contours is not of much practical importance.

20. The Theory of Instantaneous Hydrograph for Non-Uniform Rainfalls

The theory of instantaneous hydrograph has, hitherto, applied only to uniform rainfalls in the process of analysis. A uniform rainfall has to be secured in order to derive an instantaneous hydrograph from its observed hydrograph. However, so long as the latter has once been obtained, time contours can be located on the map so that any non-uniform rainfall can be applied to derive its hydrograph by the process of synthesis.

The methods heretofore described are sufficient for all practical purposes in view of the fact that only scanty rainfall stations are available in a drainage basin. Isohyetal lines at different times can hardly be drawn for a given non-uniform rainfall, that requires many automatic rainfall stations fairly scattered in the basin. However, after a decade or two, if the theories and methods here presented have been popularly accepted, competent engineers may feel the necessity of establishing many automatically recording rain gages in a basin, wherefrom it may be found that there is no uniform rainfall in the world except for very small areas. It may then be realized that the present theories and methods are not practicable in application, since no rainfall is uniform and hence an instantaneous hydrograph cannot be obtained. Consequently, it is necessary to extend the theory of instantaneous hydrograph to the field of non-uniform rainfalls.

Let several sets of isohyetal lines for different times be given for a storm passing over the drainage basin. And suppose, the resultant time contours have been located on the map, such as one of Fig. 9. Then, for any time t_1 within the storm period, we have,

$$Q_1 = \int_{(t_1 - \Delta t)}^{t_1} \left[f_0 \cdot \frac{dR}{dt} \right] dt + \int_{(t_1 - 2\Delta t)}^{(t_1 - \Delta t)} \left[f_{\Delta t} \cdot \frac{dR}{dt} \right] dt$$

$$+ \int_{(t_1 + 3\Delta t)}^{(t_1 - 2\Delta t)} \left[f_{2\Delta t} \cdot \frac{dR}{dt} \right] dt \quad \cdots\cdots \quad + \int_0^t \left[f_{(t_1 - \Delta t)} \frac{dR}{dt} \right] dt \qquad (27)$$

where the f's and $\frac{dR}{dt}$'s in the brackets are the functions of the instantaneous hydrographs occurred at the times as indicated by the subscripts and of the intensities of rainfall respectively, both to be evaluated at the time as indicated by the limits of integrations. The latter limits denote the times designating the time contours. Δt is a small increment of time. The notations of the limits of integration follow those of the last section. Now, as Δt approaches zero, it can be shown that, for any time t within the storm period, Equation (27) becomes,

$$Q = \int_0^{t_1} \left[f_0 \frac{dR}{dt} \right] dt \qquad (28)$$

which shows that only the initial instantaneous hydrograph enters into the resulting equation.

Similarly, for any time t after the storm period, it is obvious that

$$Q = \int_{t_1 - D}^{t_1} \left[f_0 \cdot \frac{dR}{dt} \right] dt \qquad (29)$$

Equations (28) and (29) are useful in the synthesis of hydrograph. For the analysis of hydrograph, the following equations found from differentiating (28) and (29) can be applied.

When $t < D$,

$$f_0\,(t_1) = \frac{dQ}{dt} \Big/ \frac{dR(t_1)}{dt} \,. \qquad (30)$$

When $t > D$,

$$f_0\,(t_1) = \frac{dQ}{dt} \Big/ \frac{dR(t_1)}{dt} + f_0(t_1-D)\,\frac{dR(t_1-D)}{dt} \Big/ \frac{dR(t_1)}{dt} \,. \qquad (31)$$

The actual computations can be carried out in a similar manner as described in Part II. Here, the time contours have to be approximately assumed first, so that $\frac{dR(t_1)}{dt}$ on any time contour t_1 can be determined; and finally these time contours may be revised.

V - Problems Involving Channel Storages

21. A Method of Eliminating the Effect of Channel Storage

A hydrograph at a given section of a river representing stream flows travelling down through the channel is modified in reaching another section due to the effect of channel storage. The problem of routing floods is to find the downstream hydrograph with the up-stream one given, or vice versa. If the upstream section instead of being a section across the stream, is taken as a long vertical one tracing along the water edges of the channel, the result of finding the hydrograph passing through such a section with the downstream hydrograph given is equivalent to eliminating the storage effect of the whole channel. As it will be seen later, this process is very useful in solution of hydrologic problems.

The method to be described is based upon the fact that for a given rainfall distribution, there is approximately a definite amount of water stored in the river channel at a given stage. This statement holds so long as the distributions of rainfall are not radically different from each other. The method will be evident if the following procedure is followed.

(1) From the many staff gages in the river channel, read the simultaneous stages from a given rainfall. Using these data together with a contour map, calculate the volumes of water stored in the channel at different stages of the stream at the gaging station. Plot the results in the G-V (Gage-Volume) quadrant. (Fig. 11).

(2) Plot the rating curve of the stream measurements at the

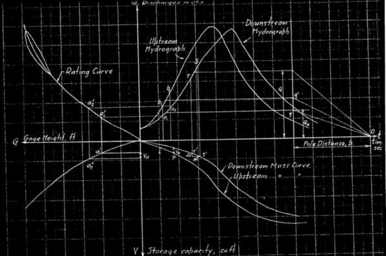


Fig. 11.- Elimination of The Effect of Channel Storage.

gaging station in the G-Q quadrant. If it varies considerably with surface slopes, plot as many curves for several slopes as may be used. In Fig. 11, only two curves are given distinguishing between the rising and falling stages.

(3) Plot the known downstream hydrograph at the gaging station in the Q-t quadrant. Integrate it according to $dV = Q\,dt$. Plot the mass curve in the quadrant V-t. This is best done graphically: select an arbitrary pole distance p on the t - axis. To integrate, for instance, the area included under the part of curve rs, assuming that the parts of the curves from t = o to r and r^1 have already been determined by the method to be described, draw a horizontal line through the mid-point of rs and join the ray to the pole. From r^1 on the mass curve, draw a line r^1s^1 vertically under r s and parallel to the ray. The point s^1 is a point on the mass curve. The reason is simple: Since by proportioning the sides of the similar triangles, $\frac{dV}{dt} = \frac{Q}{p}$, hence $dV = \frac{1}{p} \cdot Q\,dt$, in which p can be duly taken care of in the V scale.

(4) The upstream mass curve, is determined as follows: Consider a small part of the curve $a_1\,a_2$. Extend horizontal lines into the G-Q quadrant to intersect the rating curve at a_1^1 and a_2^1. Extend vertical lines to intersect the storage curve at a_1 and a_2. The vertical intercept V_a represents the water stored in the channel when discharge increases from a_1 to a_2. It must also be equal to the difference between the two mass curves. Hence draw a lines L P parallel to g o, the ray of the average discharge between a_1 and a_2, and extend it as far as a point where the vertical intercept is equal to V a.

(5) Differentiate graphically the part of the upstream mass curve just obtained to get a part of the upstream hydrograph, b_1b_2. Draw from the mid-point of b_1b_2 a horizontal line to q^1 and also the ray q^1o. Revise the part of the mass curve [illegible] to be parrallel to q^1o instead of q o. This is necessary because the slope of [illegible] represents the average discharge between b_1 and b_2 rather than a_1 and a_2. The steps (4) and (5) may be repeated as many times as seems necessary for a close check. The upstream mass curve and hydrograph are thus gradually extended.

The method, though sound theoretically, can hardly be applied in practice at present, because there are not many, if any, staff gages erected along a stream channel. The storage curve, therefore, is not known or else is not accurate.

22. On the Horton's Method of Correction For Channel Storage.

Robert E. Horton in his publication: "Surface Runoff Phenomena" page 33, describes a method of correction for channel storage. The method assumes that the point of inflection on the recession side of a hydrograph lies at or close to, the point where direct surface runoff ends, and thereafter, the flow at the outlet derives wholly from the channel storage. Upon this basis, he fixes the relation between river discharge and volume of storage which is considered as definite for the same rainfall distribution.

To discuss whether the method is justified or not, we may divide it into three statements: (1) That a hydrograph of flow derived wholly from storage cannot be convex upward, (2) that the point of inflection on the recession side of a hydrograph lies at or close to the point where direct surface runoff ends, and (3) that the relation between discharge and storage is definite for the same rainfall distribution.

The first statement is true, but the proof given by Horton has not, however, been well done. Horton remarks that: "For storage flow alone, the ratio of the discharge rate to stream stage continuously decreases. The limit of this ratio is $\frac{dQ}{dG}$, which is the slope of the hydrograph. This will always be a decreasing quantity for ordinary types of outflow from storage and hence the portion of the hydrograph representing outflow from storage alone will be concave upward, as is required by this condition." There, Horton does not actually prove it although he attempts to. A proof requires that it be shown that for storage flow alone, $\frac{d^2Q}{dt^2}$ is negative, so that $\frac{dQ}{dt}$ is decreasing. We have to set a relation between the average velocity V and gage G, and another between G and the area of section, A. Thereby apply mathematical means accordingly to find $\frac{d^2Q}{dt^2}$. However, an expedient way to prove this statement is by use of the instantaneous hydrograph. The latter represents the relation between $\frac{dQ}{dt}$ and t, and $\frac{dQ}{dt}$ is always decreasing on the recession side. The statement is readily proved.

The second statement that the point of inflection on the recession side of a hydrograph lies at or close to the point where direct surface runoff ends, is not correct. Horton has accepted it without proof. It is true that the storage flow gives a hydrograph concave upward, but it does not infer that all parts of the hydrograph that concave upward on the recession side representsflow from storage. This is a manifest mistake in the fundamentals of logic. Evidently, there are trickle flows and even overland flows after the point of inflection. We can imagine that some surface runoff near the head waters arrives at the outlet after the time of the maximum channel storage is reached. Consequently, it is not possible to locate the point where surface runoff ends. All that we can say is that this point must be somewhere to the right of the point of inflection rather than on the convex side of the recession hydrograph.

The third statement that the relation between discharge and storage is definite for the same distribution of rainfall has already been discussed in the last section. The amount of water V, stored in the channel at discharge Q can then be found by integrating the recession hydrograph from Q = 0 to Q. The relation between Q and V is thus obtained.

Following the definite Q-V relation, it is obvious that the relation between Q and t should also be definite. However, plottings of seveeal recession curves with the ground water flows deducted always show that they only coincide at very low stages but diverge at high stages. This divergence is attributed mainly to the difference in trickle flows where they are still active at high stages. The reason can be explained as follows: Although the range of discharges can well serve as a measurement of channel storage, it is of no avail for measuring waters stored in trickles or tributaries. Due to the effect of surface slopes on the S-Q relations, the same stage may represent actually different discharges. A steeper surface slope possesses longer stretches along the channel up the headwaters so that the effect of trickle flows in supplying the channel flows is more prominent. On the other hand, for flat surface slopes, the channel may not receive any trickle flow in the upstream stretch.

Consequently, in finding the [illegible]-V relations, it is advisable to plot several recession hydrographs each with ground water inflows eliminated and select from the divergent curves the steepest one in which, of course the trickle flows are least involved.

23. A Method of Finding the Ground Water Depletion Curve in The Observed Hydrograph

In hydrologic investigations and especially in quantitative studies of factors of the hydrologic cycle, it becomes desirable to separate runoff into its surface and ground water components. The efforts to make such separation are met with many practical difficulties and complexities. Some progress has been made by various investigators in the development of the various methods of separation, but none of them so far has been recognized as entirely satisfactory. In view of the importance of this problem to the theories and methods hitherto presented, a method of determining the ground water flows in an observed hydrograph is hereby proposed.

Fig. 12 shows an observed hydrograph with ground water flows

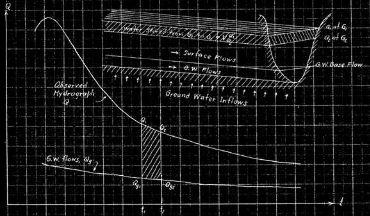


Fig 12 Separation of G.W. Flows

Q_g located. A part of the channel is also sketched. When a flow Q_1 at t_1 changes to Q_2 at t_2, the loss in the volume of stored water is $V_{Q_2}^{Q_1}$ shaded as shown in the sketch of the channel. During this interval, t_1 to t_2, the total amount of water drained out of the channel is equal to $\int_{t_1}^{t_2} Q\ dt$. Meanwhile, the ground water seeps into the channel and supplies the stream flows continuously. As the ground water flow maintains a rather uniform rate for long ranges of time, being not much affected by rainfalls, therefore, it is justified to assume that its rate of inflow into the channel is equal to the rate of outflow through the gaging station. The total amount of ground water inflow is then equal to $\int_{t_1}^{t_2} Q_g\ dt$. Consequently, $\int_{t_1}^{t_2} Q\ dt - \int_{t_1}^{t_2} Q_g\ dt$, represented by the shaded area of the hydrograph (Fig. 12) is the loss in the volume of stored water $V_{Q_2}^{Q_1}$ as shown shaded in the sketch of the channel, or,

$$\int_{t_1}^{t_2} Q_g\ dt = \int_{t_1}^{t_2} Q\ dt - V_{Q_2}^{Q_1} \qquad (32)$$

As described in the previous sections, the amount of water stored in the channel, $V_{Q_2}^{Q_1}$, is definite for the given discharges Q_1 and Q_2. If the Q-V relation for the channel has been determined, the values of $\int Q_g\ dt$ can be computed from Equation (32). After differentiation, the hydrograph of the ground water flows Q_g is thus determined.

The question arises, how can the Q-V relation first be determined? In finding this relation it is necessary to select a hydrograph fulfilling the following conditions: (1) The rainfall is quite uniform at least in the upper basin. (2) The hydrograph begins with a very low ground water, and ends at about the same low ground water flow. (3) There is no intervening rainfall of any considerable amount. It is preferable to select a hydrograph best fulfilling the above conditions rather than to take average of many hydrographs that do not follow these conditions closely. Such a picked hydrograph can be taken as the basic data for determining the Q-V relation.

Equation (32) also shows that recession hydrographs should be different for different ground water base flows. The Q-V relation should be found only when the ground water flow deductions have been made. Upon a higher base flow, the observed hydrograph takes a longer time to fill up a same amount of channel storage between two stages than upon a lower base flow. Therefore, a hydrograph with higher base flow always has a flatter slope when other conditions remain the same.

24. A Method of Separating Recession Curves

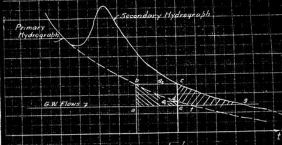
A recession curve, as the name implies, is the descending limb of a hydrograph of stream flow, including both surface runoff and ground water runoff, as it recedes from a peak downward to the point of zero surface runoff. Very often the hydrographs are characterized by peaks in such rapid succession that only rarely does the surface runoff have a chance to drain out. Such peaks complicate the whole problem and make impossible the determination of surface flows due to each single rain. Yet oftentimes a hydrograph fully possesses all properties as required for investigations by instantaneous hydrographs, etc., but is all ruined by the presence of a succeding peak. The importance of devising a method to separate recession hydrograph should not, therefore, be overlooked.

Some efforts in solving this problem have been made by many investigators, but just as those efforts toward the finding of ground water flows, there is not a single method of approach for separating recession curves, so far, that is theoretically sound for general use. An arbitrary prolongation of the primary recession curve depends so much upon the personal effects that it should never be resorted to for an important investigation.

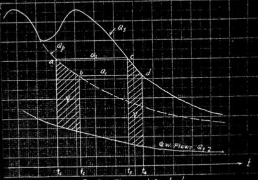
The method hereby proposed may be described under two cases according to the conditions of ground water flows.

(I) When the hydrographs begin and end at about the same low base flows so that it is justified to consider that the ground water flows are constant or the base flow hydrograph is horizontal in the related period.

Fig. 13, I, shows a primary and a secondary hydrographs, with horizontal ground water flows. The dotted recession curve is that required. There are two conditions to be fulfilled: (1) From the conclusions of the last section, the same ground water flow gives a definite recession curve, so the dotted recession curve can be determined by simply shifting horizontally the known recession curve of the secondary hydrograph to the left so as to join the known part of the primary hydrograph. (2) The storage between Q_1 at d and Q_2 at b or e can be expressed in two ways: either



I - G.W. Flows Constant



II - G.W. Flows not Constant

Fig. 13 - Separation of Recession Curves

the area abde or the area cdfg as shown in the last section. So it is required that the point b should be so located (d is given) that these areas are equal. However, it can be shown that this condition is nothing more than a correlary of the last one. From condition (1), $\overline{b\ c}$ equals $\overline{f\ g}$. If the area bcd is added to both of the shaded areas, then, b c g f d = $\overline{b\ c}$ x $\overline{c\ e}$, also, a b c d e = $\overline{b\ c}$ x $\overline{c\ e}$. Therefore, b c g f d = a b c d e, or a b d e = c d f g. Q.E.D.

(II) When the ground water flows are not uniform or the base flow hydrograph is not horizontal in the related period, - Fig. 13, II shows the primary and secondary hydrographs and the ground water base flows.

Applying Equation (32), we have,

$$\int_{t_3}^{t_4} Q_g\,dt = \int_{t_3}^{t_4} Q_s\,dt - V\Big|_{Q_1}^{Q_2}$$

$$\int_{t_1}^{t_2} Q_b\,dt = \int_{t_1}^{t_2} Q_g\,dt + V\Big|_{Q_1}^{Q_2}$$

With the Q-V relation previously determined as described in the last section, Q_g, the ground water base flows can be located from the first equation. Knowing Q_g, we can then determine Q_p from the last equation. Again, if we add together these two equations, we have $\int_{t_1}^{t_2} Q_p\,dt = \int_{t_3}^{t_4} Q_s\,dt$. Therefore, the areas shaded in Fig. 13, II must be equal. In locating the recession curve, after the point a has been determined, b should be so located that the shaded areas above the base flows are equal. The curve is thus traced out from the known intersection point gradually downward.

VI - Details of Solution - with an Illustrative Example

25. Selection of Data for Analysis

The West Branch of Salt Fork Basin at Urbana, Illinois has been chosen to illustrate the details of solution of the problems involving rainfall and runoff correlations. A map of the area is shown in Fig. 14. It has a drainage area of 60.50 square miles. Measurements of all the hydrologic data were conducted by The Engineering Experiment Station, University of Illinois, in cooperation with The Bureau of Public Roads, United States Department of Agriculture, and supervised by Professor W. D. Pickels of the University of Illinois. The data are contained in Bulletin No. 252 of the Engineering Experiment Station, from which those used in the present studies are selected.

As shown in Fig. 14, standard rain gages were established at Thomasboro, Mt. Vernon Church and Staley by the Experiment Station, and at Urbana by the United States Weather Bureau. The gage at Urbana is an automatic tipping bucket gage which gives a continuous record, except during the winter months, when its use is discontinued on account of freezing temperatures, and the record consists of two daily readings of a standard gage. The standard gages at the other stations were read daily by paid observers at 6 P.M., and in most cases the times of beginning and ending of the storm were recorded. The records at Thomasboro apply to 47% of the area, those of Mt. Vernon to 35%, Staley to 1% and Urbana to 17%.

The staff gage at Urbana was read twice daily by paid observers,

Fig. 14 - Time Contour Map

at 6 A.M. and 6 P.M., during normal periods of flow, and during flood flows several extra readings were taken daily. Discharge measurements were made according to the standard methods. The results are plotted as rating curves. (Fig. 15) As discharges at the same stage differed considerably in the summer and fall and in the winter and spring, due to the growth of weeds in the channel, two rating curves were given under extreme conditions.

For selecting data for analysis, the required conditions of rainfalls and hydrographs have been given in Section 6, Part II and Section 20, Part III. The procedure is as follows: (1) Look at the hydrographs given in Figs. 7 to 10 of Bulletin 232, Engineering Experiment Station, for those that have single peaks or peaks not too close to each other. (2) For these hydrographs, examine in Table 5 of the bulletin, for uniformity and isolation of the corresponding rainfalls in the nearby Weather Bureau Stations. Of course, small rains falling afterwards do not have serious effects on the hydrographs. (3) From diagrams prepared by Professor G. W. Pickels (not given in the bulletin) showing rainfall records of the individual stations, examine the uniformity of these qualified rainfalls. It is required to select two such rainfalls, occurring in the same month, with considerable difference in intensities.

Two of the rainfalls conforming the above requirements are given below:

March 13, 1925	Thomasboro 47%	Mt. Vernon 35%	Urbana 17%	Stuley 1%
6 A.M. - 6 P.M.	2.00"	1.97"	2.51"	2.60"
6 P.M. -12 P.M.	0.15"	0.15"	0.16"	0.22"

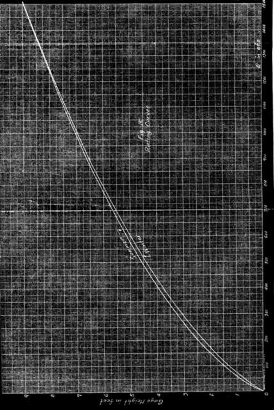


Fig. 15
Rating Curves
Winter
Summer
Gage Height in feet
0
1
2
3
4
5
6
7
8
9

(For the first 12 hours, intensity is taken as 0.167"/hr; for the last 6 hours, intensity is 0.025"/hr.)

March 18, 1925

6 A.M. - 6 P.M.	1.04"	1.04"	1.37"	1.45"
6 P.M. - 9½ P.M.	0.14"	0.08"	0.10"	0.15"

(For the first 12 hours, intensity = 0.092"/hr., for the last 3½ hours, intensity = 0.029"/hr.)

The corresponding hydrographs and rainfalls are both shown in Fig. 16. The minimum temperatures before March 15th had been below 32°F.

26. Study of the Data

The rainfall data of the Salt Fork area as recorded by the Engineering Experiment Station, University of Illinois, has one important feature superior to those recorded by the ordinary Weather Bureau Stations, i.e., the beginning and ending of a storm are mostly known. This makes it possible to estimate the approximate intensity of rainfall. Nevertheless, for records of the ordinary rain gages, the durations and intensities can be determined by the method described in Section 7, Part II, and also by a method to be described later. Consequently, the present example is applicable also to the ordinary available data.

Both rainfalls selected are composed of two different intensities, a heavy one followed by a light one. The light ones will be neglected in the following computations. This introduces an error which, however, will not be considerable on account of their small amounts. Therefore, the rainfall intensities are

0.167 and 0.092 inch per hour, and the durations are both 12 hours.

Fig. 16 shows that the hydrograph due to March 18th rainfall superimposes upon the recession hydrograph due to the March 13th rainfall. They have to be separated. Before doing this, however, those irregularities in any portion of the hydrographs should be carefully studied as to their sources of errors. If there are errors, due corrections should be made.

In order to illustrate certain characteristic features of the hydrographs, another hydrograph due to the rainfall of March 28, 1924 is given in Fig. 17 for comparison. Three irregular features are shown: (A) hump, (B) platform, and (C) a mis-record. Generally, the formations of a hump or platform may be due to one of the several possible reasons:

(1) The lowering of temperature below 32°F. within two or three days after the end of the rainfall causes the thin layer of overland waters to freeze before draining into the streamlets. Thus in Fig. 17, minimum temperatures were below 32°F. from March 30th to April 3, 1924, just the time immediately after the end of rainfall at March 29th. Therefore, part of the overland water freezes and is detained on the ground just like temporary pondage, thereby causing a general reduction of discharges observed at the gaging station during these days. Near the end of this period, the ice thaws rather quickly, causing an abrupt increase of discharges, and hence resulting in a hump. If this temperature effect were absent, the hydrograph would look like the curve shown dotted. (Fig. 17).

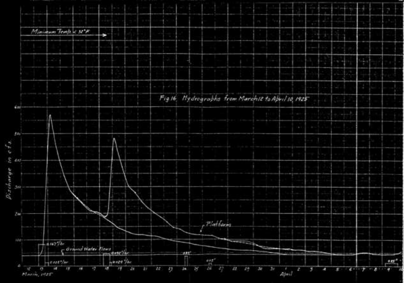


Fig 16 Hydrographs from March 12 to April 10, 1925

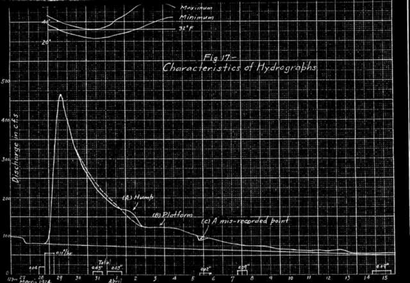


Fig. 17 — Characteristics of Hydrographs

Freezing during the night and thawing during the day may cause a small hump on the hydrograph, the range of which will, evidently, be within 24 hours.

(2) The irregularity of a river section also makes the irregularity of the appearance of a hydrograph. A sudden widening of the channel at a stage affords a sudden increase in the available amount of channel storage during rising stages. On a recession hydrograph, when discharges reduce to this stage, evidently, it takes longer time to lower one foot of stage at this point than ordinarily where there is no sudden widening. Consequently, a platform results, such as the one shown in Fig. 17 at (3).

Platforms formed due to this reason should, of course, prevail among all hydrographs at the same stage. Thus, the platform in Fig. 17 occurs at a discharge equal to 123 c.f.s., that in Fig. 16 at 116 c.f.s. A slight discrepency is probably due to the effect of surface slopes on observed discharges.

Consequently, these platforms are of general consequences, and hence should by no means be corrected as in the case of a misrecorded point.

(3) An abrupt change of the rate of infiltration along those time contours with respect to their neighboring ones from where the water contributes to the outlet as indicated by that part of the observed hydrograph is another possible reason for an irregular appearance of the hydrograph. Thus, if the ground is a rock outcrop in the neighborhood of some time contours, the rainfall excess there, is about equal to the intensity. When those waters with a

greater rate of flow, arrive at the outlet at the times as designated by the time contours, the hydrograph will evidently show an increase of discharge, or may be a hump. Similarly, a pervious ground in a certain zone of time contours causes a steeper slope of the hydrograph at those times.

From the above discussions, it can be concluded that if we carefully search all the possible sources of irregularities, there is no feature of a hydrograph that cannot be explained. The importance of such a study should never be overlooked.

The hydrographs in Fig. 16 are thus duly corrected after comparisons with other hydrographs. The general principle is to reserve all features that are common to all hydrographs and to correct those which are not. The dotted lines in Fig. 16 are those corrected.

The ground water flows in the example are extremely low, only 40 c.f.s. on March 12th, corresponding to a gage of 0.60 foot, and 45 c.f.s on April 6th, or 0.66 foot gage. The base flow is therefore, very uniform. These fulfill the conditions of Case I, Section 28, Part V for locating the separation line of the two hydrographs. As the base flow is horizontal, the separation line is obtained by simply shifting horizontally a constant phase of time. A phase of 4.4. days is used.

27. Adjustment of the Rainfall Records

The accuracy of rainfall records in representing a storm over the whole drainage area has apparently not heretofore been carefully noticed. In the present example, there are four rain gages within or nearby the area. These gages are not evenly distributed, since each applies for different percentages of area. Even if we consider that each of them receives an equal weight, i.e., 25%, which is the ideal, the total water receiving area of the four gages, at 0.35 square foot each, is only 1.40 square feet. On the other hand, the drainage area is 60.5 square miles, or 1.68 x 10^9 square feet. The ratio of the former to the latter is more than one to a billion. What has been done is to record the rainfall over each billion square feet of area by less than one square foot receiving area of the rain gage. The degree of accuracy of such recorded data needs no further explanation.

From these considerations, it is evident that the probable errors of rainfall records are far greater than those of the discharge records. It is the former that we should strive to improve, in order to be consistent with the latter, as far as the relative accuracy of the records is concerned. This is, obviously, a pertinent question in the studies of the rainfall runoff correlations. However, in the present situation, more reliance should always be laid on the discharge records than on the rainfall records. In other words, the latter should be so adjusted as to be consistent with the former.

As the two hydrographs selected are supposed to follow the requirements of Section 20, Part III, it would be expected that the total amount of water lost as indicated by the total amount of rainfall minus that of the total surface flow should be the same for both rainfalls. The total amount of rainfall is equal to the depth of rainfall multiplied by the drainage area. Thus, one inch deep on 60.5 square miles equals 60.5 x 2,323,200 cubic feet, or 39,100 c.f.s. x hr. The total amount of surface flow is given by the area of the observed hydrograph above the base flow. The results are as follows:

Depth of Rainfall, D, used for effective intensity	Amt. of Ppt. R = D x 39,100	Amt. of surface Flow V, from area of hydrograph	Amt. of Water Losses L = R-V c.f.s.x hr.
Inches	c.f.s. x hr.	c.f.s. x hr.	
2.0	78,200	50,700	27,500
1.1	43,000	25,100	17,900
0.9	35,000	25,600	9,600

in which the last line represents values for the differential hydrograph.

The results show that the amount of water losses in the two cases has a discrepancy of 9,600 c.f.s. x hr, which is, of course, also the difference between the computed amount of differential rainfall excess (35,200) and the area of the differential hydrograph (25,600). At first glance, this discrepancy would appear very great, but a close examination of the following sources of errors will show that it is not.

(1) A little error in the recorded depth of rainfall has considerable effect on the total amount of rainfall on the area. Imagine just an error of 0.1 inch depth means 3,910 c.f.s. x hr. error in the total amount of precipitation. The 9,600 c.f.s. x hr. discrepancy is only 0.245 inch, which, if equally divided among the two rainfall records, is only 0.122 inch error for each.

(2) As described in Section 20, Part III, the absorptive power of the ground at the beginning of rainfall is so high that it takes all rain that drops on it. Therefore, the amount of water losses in the two rainfalls will not be the same, the heavier rainfall always losing more than the lighter, as is the present case. To take care of this matter, the origins of the hydrographs should be begun at the time when acting runoff begins. In the first two or three hours, the rainfalls should not be taken into account. Thus, if the first three hour rainfalls are discarded, the primary rainfall will be only 1.50 inches, the secondary only 0.825 inch, making a differential rainfall of 0.675 inch. Originally, this is 0.90 inch, showing a reduction of 0.275 inch, which is more than 0.245 inch, the amount required for balancing the water losses in the two cases. Actually, of course, three hours are too long for the time required before acting runoff begins.

(3) In this example, the observed hydrographs are superimposed upon one by another, and have been separated. Suppose we move the separation line 13 c.f.s. up, the area of the primary hydrograph will be increased by 4,800 c.f.s. x hr. and that of the secondary decreased by the same amount, thereby the amounts of water losses are balanced. After all, 13 c.f.s. is only a very small percentage of the average discharges, compared with the accuracy of measuring discharges and recording gages. If such an adjustment is made, the results will be as follows:

Depth of Rainfall Inches	Amt. of Ppt. c.f.s. x hr.	Amt. of surface Flows c.f.s. x hr.	Amt. of Water Losses c.f.s. x hr.
2.0	78,200	55,500	22,700
1.1	43,000	20,300	22,700
0.9	35,200	35,200	0

The above discussion manifestly shows the fact that the amount of water losses as computed from the difference of rainfall and runoff is not at all reliable. A slight error in either of the three factors mentioned causes large discrepencies in the results. Such a discrepancy should by no means discourage an investigator. As the errors arising from the rainfall records are more probable than those from discharge readings, the following adjustments are made, mainly according to the reasonings described in the factor (2):

Depth of Rainfall, Inches Amt. lost before Run-off	Amt. effective for runoff	Amt. of Ppt. c.f.s x hr.	Amt. of surface flows, c.f.s. x hr.	Amt. of Water losses c.f.s. x hr.
0.33	1.67	65,200	50,700	14,500
0.09	1.01	39,500	25,100	14,400
	0.66	25,700	25,600	100

Here, the amount of precipitation for the primary rainfall during the first two hours, 0.33 inch, is considered as all lost by absorption before active runoff began. For the secondary rainfall, when the ground was comparatively wet at the start and the rainfall intensity smaller than the primary, this amount should have been much smaller. Only the precipitation in the first hour is considered lost by absorption before active runoff began. After such an adjustment, the results show that the amounts of water losses are practically balanced.

The recorded durations of rainfalls are both about 12 hours as given in Section 29. However, they are both doubtful. The light rainfall immediately following may cause some errors. After this adjustment is made, the effective durations should be 10 and 11 hours for the primary and secondary rainfalls respectively. But in view of the doubtfulness of the records, both are taken as 10 hours to simplify the later computations. The intensities are then,

I_1 = 0.167 inch per hour

II_2 = 0.101 " " "

28. Determination of the Differential Hydrograph

As hitherto stated in Part II, an observed hydrograph at the gaging station is contributed from rainfall excesses that vary with times and locations of the basin. In order to analyze it by the method of instantaneous hydrograph, a differential hydrograph must be found due to a uniform rainfall excess. This can be done by superimposing two observed hydrographs of different intensities upon one another on a common time axis, so that the rates of water losses occurring at the same times and on the same time contours can be automatically eliminated. Nevertheless, two such hydrographs of different intensities always have sets of time contours different from each other. The velocities of flow in the case of the greater hydrograph are always faster than those of the smaller one.

Therefore, it is necessary to correct one of the two such that they possess the same set of time contours.

Fig. 18 shows the method of correcting the secondary hydrograph for the present example. The secondary hydrograph is first plotted. (S. $H._0$) Next the primary hydrograph is plotted in such a position with respect to the secondary, that their common recession hydrograph coincides. Then the primary recession curve is deducted from the secondary hydrograph, S.$H._0$, to obtain the net runoffs due to the secondary rainfalls. (S.$H._1$). It is seen that the two have different starting points, the secondary lagging behind the primary by about 13 hours. The total flood period of the primary is about 19 days, that of the former about 18½ days, the difference being very small. The hydrograph, S.$H._2$, corresponding to the secondary, S.$H._1$, but following the time contours of the primary, is found as follows. Let t_1 and t_2 be the corresponding times and Q_1 and Q_2 the corresponding discharges of any point on S.$H._1$ and S.$H._2$ respectively. Then t_2 is found by multiplying t_1 by 19/18½, and Q_2 from multiplying Q_1 by 18½/19. The idea is to lengthen the t-axis while lowering the Q-ordinates inversely in proportion at every point, so that discharges from the area included between a time contour and the outlet as represented by the areas under the hydrographs from t = 0 to t_1 and t = 0 to t_2 are equal for all corresponding points on the two hydrographs.

Strictly speaking, a recession hydrograph, when fully laid out, should be infinitely long, since the time required to wholly drain out the water from a stream approaches infinity. The half

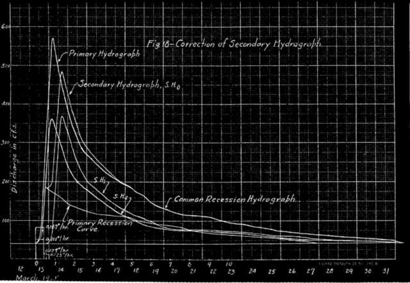
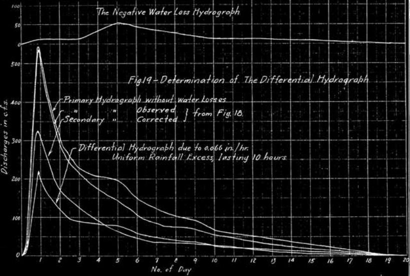


Fig 18 - Correction of Secondary Hydrograph
Primary Hydrograph
Secondary Hydrograph, S.H0
S.H1
S.H2
Common Recession Hydrograph
Primary Recession Curve
Discharge in c.f.s.
600
500
400
300
200
100
0 1 2 3 4 5 6 7 8 9 10
12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
March


The Negative Water Loss Hydrograph
Fig.19 - Determination of The Differential Hydrograph
Primary Hydrograph without water losses
" " Observed
Secondary " Corrected
from Fig. 18.
Differential Hydrograph due to 0.066 in./hr.
Uniform Rainfall Excess, lasting 10 hours
Discharges in c.f.s.
No. of Day
100
50
0
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

day or 13 hour difference is negligible for very long bases. Therefore, the ratio 18½/19 is rather arbitrary. The effect of this approximation will be observed later.

The primary hydrograph observed and the secondary hydrograph corrected in Fig. 18 are reproduced in Fig. 19. From them the differential hydrograph is obtained, which is due to a uniform rainfall excess of 0.066 inch per hour rate and 10 hour duration. The area included under the hydrograph should be 25,800 c.f.s. x hr. (0.66 x 39,100), but it is found 10% less, due to errors in computations. It is seen that a platform is present, which, if traced back carefully, will be found due to the original platforms in the hydrographs.

The ordinates of the differential hydrograph are then multiplied by $\frac{I_1}{I_1 - I_2}$, or 2.53, to obtain the primary hydrograph without water losses. Deducting the observed primary hydrograph from the latter, we obtain the negative water loss hydrograph as shown. It shows a peak at the end of 5 days, which also results from the platforms.

For an immediate test of the results so far obtained, a uniform rainfall of about the same duration is used in synthesis. It occurred on March 28, 1924, and has the following records:

	Thomasboro 47%	Mt. Vernon 35%	Urbana 17%	Staley 1%
11 A.M. to 6 P.M.	0.28"	0.28"	0.35"	1.40"
6 P.M. to 6 A.M.	1.32"	1.32"	1.03"	1.40"

The average intensity during the first 7 hours at 0.30" total is

0.04"/hr., during the next 12 hours at 1.32" total is 0.11"/hr. According to the primary rainfall used in analysis, an amount of 0.33" is lost before active runoff began. The present rainfall has the same initial conditions, i.e., there was no precedent rainfall for a long time. Therefore, the 0.30" rainfall during the first 7 hours is probably all lost for moistening the ground. The effective rainfall can be taken as 0.11"/hr. for intensity and 1. hours for duration. The observed hydrograph has been given in Fig. 17. With ground water flows deducted, it is reproduced in Fig. 20.

The computed hydrograph with no account of water losses is found by multiplying the ordinates of the differential hydrograph by the ratio 0.11/0.066, or 1.67. As this is simply for a test, the 12 hour duration is taken as 10 hours so as to be the same as the duration of the differential hydrograph. Deducting the known water loss hydrograph, we obtain the computed hydrograph.

It is thus seen that the two hydrographs, computed and observed, are quite close together especially on the recession side. The arbitrary assumption of the 10 hour duration causes the discrepancy in the first three days. The peak flows of the computed hydrograph show too low because the duration is taken two hours shorter than it should be. However, the discrepancy in the part before the peak should be attributed to the approximations made in the analysis when correcting the secondary hydrograph to conform to the time contours of the primary. (correcting $C.H._1$ to $C.H._2$ in Fig. 18). As the secondary hydrograph begins at a much

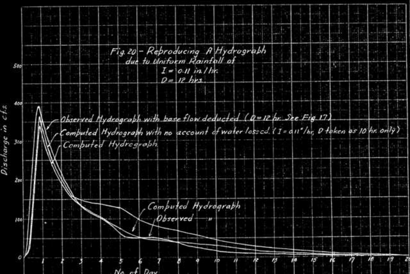


Fig. 20 - Reproducing A Hydrograph due to Uniform Rainfall of
I = 0.11 in./hr.
D = 12 hrs.
Observed Hydrograph with base flow deducted (D = 12 hr. See Fig. 17)
Computed Hydrograph with no account of water lossed. (I = 0.11"/hr, D taken as 10 hr. only)
Computed Hydrograph
Computed Hydrograph
Observed "
Discharge in c.f.s.
No. of Day
0
100
200
300
400
500
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20

higher river stage than the primary, the velocities of flow are higher in the secondary during the first day. Hence the time contours near the outlet embrace greater contributing areas for the secondary than the primary. Consequently, in correcting the secondary hydrograph to conform to the time contours of the primary, the ordinates of the former should be reduced according to the ratio of the respective contributing areas. It can be shown that this ratio is greatest at t = 0 and gradually reduces to one when the time contours in both cases possess the same area. The process of correcting S.H.1 to S.H.2 in Fig. 18 results in a differential hydrograph during the first day which is too low. The effect has been reflected in the computed hydrograph of Fig. 20, and of course, shall always exist in all reproduced hydrographs.

29. Determination of the Instantaneous Hydrograph

The instantaneous hydrograph is determined according to the method described in Sections 6 and 7. The computations are given in Table 6 and the results plotted in Fig. 21.

The area of the instantaneous hydrograph is found to be 1990 c.f.s. It should be equal to the intensity of rainfall times the drainage area, or, 0.066 x 39,100 = 2580 c.f.s. This error is mostly from computations. It arises first, from the errors in computing the differential hydrograph, as described in the beginning of the last section, and then from the rough computations of the instantaneous hydrograph. However, the writer will not revise the whole problem. These results are presented here purposely

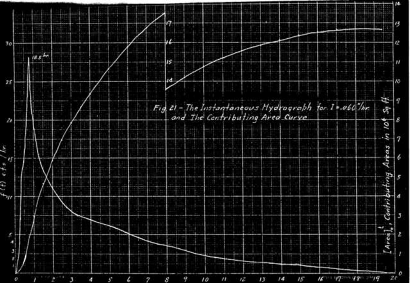


Fig. 21 - The Instantaneous Hydrograph for I = .066"/hr. and The Contributing Area Curve

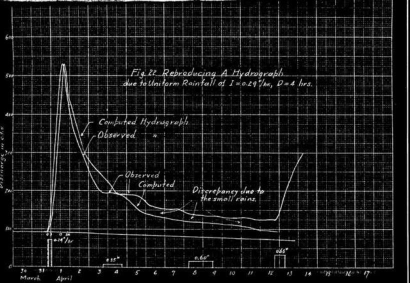


Fig. 22 Reproducing A Hydrograph
due to Uniform Rainfall of I = 0.29"/hr., D = 4 hrs.
Computed Hydrograph
Observed "
Observed
Computed
Discrepancy due to
the small rains.
Discharge in c.f.s.
600
500
400
300
200
100
0
0.29"/hr
0.35"
0.60"
0.65"
30
31
1
2
3
4
5
6
7
8
9
10
11
12
13
14
17
March
April

to show the computors the effects of rough calculations. A slight error at beginning causes serious discrepancies in the final result, mainly because the errors are cumulative rather than compensative.

A uniform rainfall is selected to test the results obtained. It occurred on March 31, 1927. The recorded data are as follows:

	Thomasboro 47%	Mt. Vernon 35%	Urbana 17%	Staley 1%
March 31, 8 P.M. to April 1, 1 A.M.	1.40"	1.40"	1.69"	1.25"

The average value is 1.45" and the duration 5 hours. Intensity = 1.45/5 = 0.29"/hr. There was no precedent rainfall for many days. In Section 31, an amount of 0.33" was lost due to infiltration before active runoff began. Here let us take ~~it to be~~ 0.29" or the first hour rainfall. So the effective duration is only 4 hours.

The method of computing the hydrograph follows that described in Section 8. The results are shown in Table 7 and are plotted in Fig. 22. It is seen that the higher part of the computed hydrograph looks like it had been shifted to the right of the observed hydrograph. This is mainly due to the uncertain time of beginning of rainfall and also the uncertain duration and intensity. The errors due to approximations in transforming the secondary hydrograph still exist in the part of the hydrograph before peak flows, as ~~was to be~~ expected. The small rainfalls occurring afterwards cause mainly the discrepancy between the two hydrographs. The errors in the instantaneous hydrograph when it was calculated have not been revealed here greatly, except on the recession side, which is, however, still uncertain. It may have been compensated by the errors in assuming the rainfall as uniform which is actually not.

Table 6a - Determining the Instantaneous Hydrograph

Day	t Hours	c.f.s.	d/dt c.f.s./2.5hr	f(t-10hr) c.f.s./2.5hr	f(t) c.f.s./2.5hr	for i = 0.066"/hr. c.f.s./hr
0		0			0	0
			2		2	1
	2.5	2				
			5		5	2
	5	7				
			18		18	7
	7.5	25	33		33	13
	10	58		0		
			36	2	38	15
	12.5	94				
			36	5	41	16
	15	130				
			36	18	54	22
	17.5	166				
			37	33	70	28
	20	203				
			22	35	58	23
	21.5	216				

Table 6b

Day	Q c.f.s.	$\frac{dQ}{dt}$ c.f.s./.25 day	f(t) for I = 0.066"/hr. c.f.s./.25 day	f(t) for I = 0.066"/hr. c.f.s./hr
20	0			
		0.3 (changed from 0.6)		
.75	0.6		0	0
		0.6		
.5	1.2		0.3	0.05
		0.6		
.25	1.8		0.6	0.1
		0.7		
19	2.5		0.9	0.15
		0.6		
.75	3.1		1.3	0.22
		0.6		
.5	3.7		1.5	0.25
		0.6		
.25	4.3		1.9	0
		0.7		
18	5.0		2.1	0.35
		0.6		
.75	5.6		2.6	0.43
		0.6		
.5	6.2		2.7	0.45
		0.6		
.25	6.8		3.2	0.54
		0.7		
17	7.5		3.3	0.55
		0.6		
.5	8.7		4.0	0.67
		0.7		
16	10		4.6	0.77
		0.5		
.5	11		5.3	0.89
		0.5		
15	12		5.9	0.99
		0.5		
14	14		6.9	1.15
		0.7		
13	17		8.3	1.4
		0.7		
12	20		9.7	1.62
		0.8		
11	23		11.4	1.9
		1.0		
10	27		14.1	2.35
		2.5		
9	37		17.7	2.95
		1.2		
8	42		20.9	3.48

		1.5		
7	48		24.3	4.05
		2.5		
6	58		29.9	4.98
		4.7		
5	77		36.5	6.09
		1.3		
4	82		41.2	6.87
		2.0		
3	90		47.6	7.93
		7.5		
2	120		65.9	10.99
		19.5		
1	198			

Table 7 - Computing the Hydrograph due to The Rainfall of March 31, 1927

t Day,	Hour	f(t) c.f.s./hr.	$\int_0^t f(t)dt$ c.f.s.	$\int_0^{t-D} f(t)dt$ c.f.s.	Q c.f.s.	Q c.f.s.	Water Loss Deductions	Net Q c.f.s.
		I = 0.066"/ hr.				I = 0.29"/ hr.		
0		0	0		0	0	0	0
	4	2.3	5	0	5	22	2	20
	8	11	31	5	26	1114	4	110
	12	15.5	84	31	53	233	6	227
	16	22	159	84	75	330	8	322
	18.5	28(peak)	222					
	20	24	261	159	102	448	10	438
1		19.8	348	261	87	382	12	370
	12	13.6	548	481	67	295	12	283
2		11.0	696	647	49	215	12	203
	12	9.4	823	782	41	180	12	168
3		7.9	927	892	35	154	12	142
	12	7.3	1018	988	30	132	22	110
4		6.9	1103	1075	28	123	23	100
	12	6.5	1183	1156	27	119	44	75
5		6.0	1259	1234	25	110	53	57
	12	5.5	1329	1306	23	101	51	50
6		5.0	1393	1372	21	92	44	48
	12	4.6	1451	1432	19	83	39	44
7		4.1	1504	1487	17	75	35	40
8		3.5	1595	1580	15	66	28	38
9		3.0	1674	1661	13	57	20	37
10		2.4	1739	1728	11	48	14	34
11		1.9	1792	1783	9	40	13	27
12		1.6	1835	1828	7	31	13	18

---

30. Determinations of the Time Contours

The time contours (Fig. 14) are located by the method described in Section 10. However, the work is very rough, and the step (8) of Section 10 has not been followed. The purpose is merely to show the reader how they look. In view of insufficient rainfall stations, it is not desirable at all to locate them accurately. With the assistance of the map in front of him, the reader would understand better the principles as outlined in Part II.

Table 8 gives the calculations of the contributing areas by means of Equation (16). As noted in the last section, the instantaneous hydrograph is erroneous and has not been revised. Accepting it as correct, the writer changes the intensity of rainfall so as to be consistent with the area of the instantaneous hydrograph. The new intensity is 0.051"/hr. (= 1990/39100). Since the intensities here are expressed in inches per hour instead of per day, the constant in Equations (16) should be duly changed: $\Big[\text{Area}\Big]_0^t$ Sq. ft. = $\frac{12\text{x}3600}{0.051}$ q_t, or, = 0.848 x 10^6 q_t. The results of Table (8) are plotted in Fig. 21. By means of this curve, the times of time contours can be readily be found corresponding to the known contributing areas measured from the map.

It is seen from Fig. 14, that the time contours may coincide, though never cross each other, such as those of 1.2, 1.8 days with 21.5 hours. This arises from the presence of a divide between the two tributaries flowing into the main stream.

The peaks of the hydrographs of this basin always arrive at 21.5 hours after the beginning of rainfalls, because the peak of an instantaneous hydrograph is proportional to the longest time contour. (Equation (13), Part II). Thus, the lengths of the time contours 8.5 hrs., 13.8 hrs., 1.2 days, 1.8 days, are all shorter than that of the 21.5 hr. contour. Although the 2.9 days, 8.3 days contours near the headwaters are longer than the latter, but their velocities, $\frac{ds}{dt}$ (Equation (13)) in crossing them are so slow as to make the product, f(t), small. The very divide between the two

Table 8 - Computations of the Contributing Areas

t Day	$\int_0^t f(t)\,dt$, c.f.s. for I = 0.051"/ hr. Values from Table 7	$\text{Area}\Big]_0^t$. 10^6 sq. ft. $= .848 \times 10^6 \int_0^t f(t)\,dt$
0	0	0
.25	18	15
.5	84	71
.75	220	187
1	348	295
.5	548	465
2	696	590
.5	823	697
3	927	786
4	1103	936
5	1259	1067
6	1393	1182
7	1504	1275
8	1595	1352
9	1674	1420
10	1739	1473
11	1792	1520
12	1835	1555
13	1871	1586
14	1902	1613
15	1928	1633
16	1950	1652
17	1967	1667
18	1979	1677
19	1986	1682
.5	1990	1690

tributaries mentioned in the above paragraph causes the 21.5 hr. contour to be controlling.

Many other drainage area characteristics, as discussed in Part II, can be revealed by a careful study of the present example. Space does not allow further discussions.

In concluding the present discussions, the writer might say that the examples presented in this paper are the only ones that he has ever worked out. No single example has ever been discarded, or the results changed because of its failure to meet the tests of the theories and methods that he has established. It is very important to realize the axiom that if a theory or method is correct, as has been proved by scientific methods of logic, this theory or method must be correct by itself. An example in applying it, is merely a test rather than a proof. Therefore, the satisfactory results as shown in the examples of this paper should not be inferred to imply that the writer's theories and methods so far established are correct, nor the negative be taken dogmatically as a disproof. Criticisms on their correctness and exactness should always be directed toward the theories and methods themselves.

Conclusions

From an engineering point of view, we are not particularly interested in the rates of evaporation, transpiration, etc., except indirectly, for obtaining the stream flows. Therefore, we are justified in grouping them under one head "the rate of water losses." The best way of detecting all physical characteristics pertaining to a drainage area is to find them directly from the hydrograph itself. Every irregular feature of an observed hydrograph works in accordance with principles that an analytical study will reveal.

In developing his theories and methods for solving hydrologic problems, the writer has drawn the following conclusions:

(1) The theory of instantaneous hydrograph, with the fundamental variables, time and space, of a natural phenomenon taken into consideration, is the one and only theory that is feasible for scientifically analyzing the hydrologic data for engineering uses.

(2) Much effort has been made, and the results are successful, in applying the theories and methods to the present available data of the United States Weather Bureau and the United States Geological Survey. Consequently, although the methods are highly theoretical, they are perfectly practical.

(3) The time contour analysis as a product of the theory of instantaneous hydrograph is the best and the only scientific method for analyzing the characteristics of a drainage area. It yields many practical applications, such as, computing hydrograph

for non-uniform rainfall, which has never been hitherto made possible; determining the velocities of overland flows in soil erosion problems; routing floods in river channels; estimating maximum peak flows at any point in a drainage area, etc.

(4) Criticisms have been made on the unit-graph method which explains the inaccuracies and deficiencies of the method.

(5) The method of differential hydrograph is recommended for taking care of the variable rate of water losses on a drainage area. The negative water loss hydrograph may not be analyzed into the instantaneous hydrographs for practical purposes. As a greater part of the amount of water lost in a storm is usually attributed to the infiltration and detention before active runoff begins, the determination of which, by working on many examples in analyses, is pertinent.

(6) The generally accepted hypothesis of a constant flood period for a drainage basin has been proved to be a fallacy.

(7) The analysis by the theory of the variant instantaneous hydrographs is an exact one, but the method proposed for synthesis is only approximate.

(8) The theory of instantaneous hydrograph for non-uniform rainfalls is a device for finding the instantaneous hydrograph from a given non-uniform rainfall. While it is not needed at present, it will be useful when the rainfall stations are abundant enough.

(9) A method of eliminating the effect of channel storage is proposed, though it is not practical due to lack of staff gages

along the main river channel. Criticisms are given to Horton's method.

(10) The method of finding the ground water depletion curve and the method of separating recession curves proposed are useful in actual solutions of problems. The accuracy of these methods, however, depends upon the accuracy of the Q-V relations which changes with the distribution of rainfall.

(11) The example given in Part VI illustrating the details of solution has met the tests of the theories and methods established. However, these satisfactory results should not be inferred to imply that the writer's theories and methods are correct, nor the negative be taken dogmatically as a disproof. Criticisms on the correctness and exactness should always be directed toward the theories and methods themselves.

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